Wave Resistance of Thin Ships and Catamarans

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Abstract

Michell's thin ship theory is implemented to yield the wave resistance of single or double-hulled vessels. The program takes offset data as input. Comparisons are made with previous computations and experiments, both for conventional single-hull ships and for the class of catamarans known as SWATH ships. The present computational procedure is shown to be at least as accurate as methods currently in use.

1 Introduction

The famous Michell integral (Michell 1898) for the wave resistance of a thin ship has not been used very widely in Naval Architectural practice. Its accuracy is sometimes questioned, but not always fairly. Comparisons of inaccurate computations with questionable experiments, etc.!

Indeed, the true Michell integral has seldom been computed for actual hulls. Mostly the computations have been for mathematically defined hulls, for which one or more of the three integrations can be done analytically. When this is not the case, we must be very careful to do the numerical integrations properly, and this is one of the present objectives.

The thin-ship approximation (i.e. small beam to length ratio) applies to multiple as well as to single hulls. There is a potential complication associated with interaction between the hulls, but this is generally ignored. The main purpose of the present report is to construct a computer program for the wave resistance of SWATH ("Small Waterplane Area Twin Hull") ships, but the same computer program also is capable of giving the wave resistance of conventional ships or catamarans.

The structure of this report is as follows. In Section 2, the existing literature on SWATH-type vessels is reviewed. In Section 3, sample computations on idealised submerged mono-hulls are used to test applicability of thin-ship theory. Section 4 contains the full details of the theory and numerical methods as programmed, and Section 5 the results.

The results (Figure 6) for SWATH ships confirm that the present program evaluates the wave resistance with an accuracy equal to or better than that of published procedures.

2 SWATH background

A SWATH ship is a catamaran with individual hulls of a "bottle"-like section, as sketched in Figure 1. That is, each of the two hulls has a fully submerged main hull of a submarine character, atop of which is a thin surface-piercing strut, the two struts being connected together by an above-water platform or structure.

A feature of this design is that the separate individual wetted hulls are rather thin by usual ship standards, the submarine hull typically having beam/length ratios of 0.1 or less. The strut is of course even thinner, with beam/length say 0.04. The draft/length ratio of the whole structure is comparable to that of a ship, say 0.1.

A natural model for this semi-submerged hull configuration (whether a single hull or a catamaran) is to split it into two components, namely the submarine hull and the strut. The latter is clearly an amply-thin body to which to apply Michell's thin-ship theory. Since the former has a cross-section that is more-or-less circular, it can be modelled as a slender body.

The actual implication of this approach on the hydrodynamics is that the flow is modelled by the sum of a line distribution of sources modelling the slender submarine (strength proportional to the longitudinal rate of change of submarine cross-sectional area) plus a plane distribution of sources modelling the thin strut (strength proportional to the longitudinal rate of change of strut thickness). Thinness and slenderness implies linearity, so the effects of these singularity distributions on the flow can be added. The effect on the wave resistance is then quadratic, in an obvious way, since it depends on speed squared.

The location of these singularities is a good point of issue. The strut model involves putting the sources on the mean plane of the strut, which is straightforward if that plane is not canted, i.e. is vertical. If canted, there should be some re-consideration of the model, but fortunately any canting is unlikly to be large enough to be significant.

If the submarine part of the hull is a body of revolution, the location of the line singularity seems "obviously" to be its centerline, and perhaps for a general cross section we would choose the line (strictly the "curve", since it need not be straight) of centroids. However, it is worth noting that there is no fundamental hydrodynamic reason, in the context of slender body theory, for preferring the centroid to any other line. If we represent a submerged slender body by a line distribution of sources, the location of that line can be anywhere within a distance comparable to the beam from the centroid. That is, it

could even lie outside the body!

This observation is related to the other fact that we should comment upon, namely that slender body theory does not distinguish between shapes of cross sections, the only input being the area of that cross section. This is good and bad! It is good, because we don't have to worry about fine details of the cross-section geometry. It is bad in part for the same reason - what if those details are really important? In particular, we suspect that those parts of the body that are nearest to the free surface are the most important. What if we take a section of fixed area, and distort it significantly in the vertical? We could even do this while maintaining the same centroid submergence. Would the wave resistance be unaffected?

Those are questions that have not been answered, and it is a pity that the literature on SWATH tends to take for granted certain slender body ideas (some even attributed to me!) without settling these issues.

The most recent and thorough study of the SWATH is that by Salvesen et al (1985). This builds upon work done by Chapman (1972, 1977) on single semi-submerged hulls. A feature of this study is that a third distribution of sources is introduced. Its role is somewhat obscure, but roughly speaking it is intended to cancel out that portion of the outflow induced by the line sources on the upper part of the submarine hull that should have been blocked by the strut. Although no numbers are given by Salvesen et al, I imagine the influence of this distribution is small. For example, one of their discussers (C. Kennell) reports computations using a similar model but without the third distribution, and the final resistance results are very close (noting the authors' reply regarding an error in their first draft computations). This matter has little to do with the catamaran character of the SWATH, and would be an issue even for single semi-submerged hulls.

A matter that is specific to double hulls is the question of hydrodynamic interaction between them. By this is meant not just the obvious wave interference effects, that are included in any theory where the separate hulls are modelled by known singularity distributions, but rather influences of one hull on the other's singularity distribution. There is only one sentence on this question in the Salvesen et al paper (p. 329), to the effect that it is being neglected. However, one of the discussers (M. Wilson) quotes a reference by Lin (1974) where this topic is discussed in considerable detail. The problem is that source distributions alone cannot accurately model two thin hulls, since sources produce and account for only locally laterally symmetric flow. But each hull induces a cross flow on the other. To cancel this out, we need not only sources, but also lateral dipoles at the second hull.

It would be an under-statement to assert only that the required dipole strength is "not easy" to estimate. The task of computing the dipole strength requires solution of an integral equation comparable to the lifting-surface equation of aerodynamics, but with a wave-like kernel that is several orders of magnitude harder to compute. At this point of time, there are probably no acceptable algorithms even for the kernel, much less to solve the integral equation. So it is not reasonable to expect interaction to be included in a SWATH model, and it is no criticism of the Salvesen et al work that it is not. Indirect evidence that it may be of minor importance is the good agreement with experiment

produced by the model without it.

Formal slender-body arguments (cf Tuck and Newman 1974) can also be used to justify neglect of interaction singularities. Thus if ϵ measures the beam of a thin hull whose draft is not also small, then the effective incidence induced at one hull by the other is also of order ϵ. Hence we must expect vortex or dipole distributions also of the order of ϵ. But if the draft is also small, of order ϵ, the cross-flow induced at one slender body by the other is of order ϵ^2, and this is accounted for by lateral dipoles of strength of order ϵ^4, two orders of magnitude smaller than the source strength, which is of order ϵ^2. Hence, although for a general wing-like catamaran hull of large draft we might need the interaction term, formally it is negligible when the draft as well as the beam is small compared to the length.

However, omission of interaction is an indication that one is not justified in including too many refinements of other types. There is little point in driving a Volvo because of its safety record, on bald tyres. I believe there are areas where the Salvesen et al model is over-refined for its purpose. In the following section, I give examples to illustrate this point.

3   Michell resistance of submerged bodies

The thin ship theory of Michell represents the ship by a centreplane source distribution proportional to its longitudinal rate of change of thickness (local beam). The only requirement for its validity is that that quantity be small. Hence the theory applies as well to submerged as surface piercing bodies.

Another way of putting this argument is that slender bodies are also thin. To a certain extent, slender body theory is superfluous. Even where a body looks more slender than thin, it is unlikely that (to leading order) modelling it by a centreline singularity achieves more than modelling it by a centreplane singularity.

Indeed, there are examples where one can prove that this is the case, and wave resistance is one of them. Thus, for surface-piercing ships, Vossers (1962) and I (Tuck 1963) derived the slender ship wave resistance formula laboriously from first principles, but Maruo (1962) was able to show very simply that it could be obtained from Michell's thin-ship formula by letting the draft go to zero!

Why use slender body theory at all? A good question to which there could be several answers. One is to dispute the evidence that thin ship theory subsumes slender body theory. Since there is no exact formula for wave resistance, one cannot say from the theoretical point of view which is the better approximation. So the slender body result (even though it is an "approximation" to the Michell result) could perhaps be more accurate as an estimate of the true wave resistance. I doubt this very much. There are elements in the Michell theory (such as exponential decay with depth) that are absent in the slender body theory, and I think should be present - although others might assert the opposite. Anyway, the evidence of comparison with experiment seems to favour Michell.

The thin and slender body theories only agree to leading order in slenderness. If one

expects to improve upon either theory by including second-order contributions, there is a case for picking the more appropriate model, but there have been very few attempts at second-order theories, and no successes to my knowledge. This is certainly irrelevant to the present review, since nothing second order has been proposed for SWATHs, or should be.

The remaining argument for slender body theory is that it is a valuable computational simplification. That is, if the body is indeed slender as well as thin, we might as well make use of that fact in some way. Certainly there is one immediate simplification in the data preparation, since the shape of cross section is irrelevant in slender body theory, but input to thin ship theory. I think this is such a minor matter as to be ignored. I feel that the composite "slender body and thin strut" approach adopted by Salvesen et al is hardly less difficult to implement than a full thin-body approach, and, when the third (controversial) body-strut source distribution is also put in, any advantage of simplicity disappears.

To test some of these ideas, I considered a family of submerged slender bodies. I chose SWATH-like dimensions of 250 feet length, 20 feet centroid submergence of all sections, and $A_0 = 400$ square feet maximum cross-section area, with parabolic longitudinal variation in cross-section area. This is enough information to compute the slender-body wave resistance, and Figure 2 shows the result for three assumed submergences H (15, 20, 25 feet) of the line of sources. That is, the middle of these is when we put the source line at the centroid, and the others are 5 feet above or below the centroid.

I repeat that it is totally irrelevant to slender body theory what is the shape of the cross sections. For example, the maximum cross section could be a square of side 20 feet. In that case, the three assumed source lines are at the centroid and at one quarter width above and below the centroid. Slender body theory asserts that a spheroid of the same 400 square feet section, i.e. with circular sections of maximum radius 11.3 feet, has the identical wave resistance. Also, the same applies for rectangles of any aspect ratio, e.g. a wide hydrofoil-like body with 40 feet width by 10 feet height, or even more interestingly a narrow parabolic surface-piercing strut of 10 feet width by 40 feet height, which is not submerged at all!

I also repeat that from the point of view of hydrodynamic slender body theory, there is no reason to prefer any of the three source depths shown in Figure 2 to any other. Centroids are guaranteed no magic hydrodynamic properties. From one point of view, the variation between the wave resistances computed using these varying choices of source line is a measure of the validity of the slender body approximation, and if there was a lot of variation, we should suspect the theory.

Now if we turn to thin ship theory, there is an influence of section shape. It is very easy to use Michell's theory (as presented in detail later in this report) to compute the wave resistance of bodies with rectangular sections and parabolic waterlines. Indeed, the parabolic surface-piercing strut was computed in Michell's original 1898 paper, and was repeated by Havelock, Wigley, and many others. Submerged parabolic struts are just as easy. Figure 3 shows results for the above-mentioned strut, square and "hydrofoil".

On the same figure, we show crosses for computations for spheroids, i.e. bodies with

circular cross sections of maximum radius 11.3 feet, with the same maximum cross-section area and centroid depth as the bodies with rectangular cross sections, and with the same (parabolic) section area variation along the length. These are not at all easy to compute, and the full computer program to be described later was used, with input offset data. Note that the non-dimensional resistance coefficient used in both Figures 2 and 3 is normalised with respect to the midship (maximum) cross-section area $A_0 = 400$.

Incidentally, the wide rectangle is part-way to a limiting result that we might have anticipated theoretically. Namely, if we consider a family of rectangular sections of fixed cross-section area and fixed centroid submergence, and let the width tend to infinity (hence height tend to zero), the result is formally identical to that given by slender body theory. That is, from the point of view of thin-ship theory, slender body theory is just the "small-height" limit. Indeed, the most convenient way to work out the slender-body wave resistance is just to run the thin-ship program for rectangles with very large width/height ratios! Not only that, but a 40 wide by 10 high rectangle is already of sufficient width/height ratio for this purpose, and the curve for the wide rectangle in Figure 3 is indistinguishable from the slender body result at depth $H = 20$ in Figure 2.

Let us summarise the conclusions. For bodies of the dimension used, there is indeed very little influence of cross-section shape on the wave resistance, at fixed cross-section area and centroid submergence. In particular, the square and circular-section results are very close to each other. Even the strut and hydrofoil have resistances very little different from each other, from the square and circle, and from the slender body results at the same centroid submergence. These differences are much less than the differences between the slender body results due to variation in the depth of the line of sources.

What to learn? For slender body theory, centroids do seem to possess some magic! That is, if we're going to use slender body theory, it is very important to put the line of sources close to the centroid. But the more important message is that we needn't use slender body theory at all if we prefer not to, since the results using thin ship theory are very close to those using slender body theory. Thin ship theory does not use a separate centroid specification, but rather inputs data about the full cross-section shape that implies it.

There seems to me to be no reason not to use thin-ship theory for the complete wetted hull, for semi-submerged bodies with SWATH-like dimensions.

4 Computational Details

According to Michell's thin-ship theory, the wave resistance of a single hull with equation $y = \pm Y(x, z)$ is obtainable by a sequence of three integrals, as follows.

First evaluate for all stations x, and all values between 0 and $\pi/2$ of a wave propagation direction θ, the integral

$$F(x, \theta) = \int Y(x, z) \exp(\kappa z \sec^2 \theta) \, dz$$

where $\kappa = g/U^2$, and the integral is in the vertical z direction from the lowest point of

the section to the waterline. The offsets $Y(x, z)$, namely half-widths at station x and waterline z, are specified as data.

Next evaluate simultaneously a pair of integrals along the ship from bow to stern, namely

$$P(\theta) = \int F(x, \theta) \cos(\kappa x \sec \theta) \, dx$$

with a similar integral for $Q(\theta)$ having sin() instead of cos(). This integral takes as input the quantity F just evaluated.

Finally, integrate the contributions to the total wave resistance R of waves of all directions θ, namely

$$R = c \int_0^{\pi/2} (P^2 + Q^2) \sec^5 \theta d\theta$$

where $c = 4pg^4/(\pi U^6)$ is a constant depending only on the speed U and $g = $ gravity, $\rho = $ water density.

The above is only a slight variation from standard versions of the original Michell integral, an integration by parts having been used to express the integral in terms of the actual offset Y rather than its longitudinal derivative Y_x, and it is written in the form of three separate integrals rather than a triple integral. The change to a catamaran (with identical thin hulls, each of width $2Y$, whose centreplanes are separated by w, as sketched in Figure 1) is almost trivial. Just multiply each of P and Q by the factor

$$W = 2 \cos \left(\frac{1}{2} \kappa w \sec^2 \theta \sin \theta \right),$$

i.e. multiply the integrand in the θ integral by W^2. This change just corresponds to shifting the location of the source distribution to $y = \pm \frac{1}{2} w$, and neglects any hydrodynamic interactions between the two hulls, as discussed earlier.

There have been few attempts to set up standard routines to evaluate this triple integral from input offset data. Perhaps for conventional ships the incentive has been absent, since Michell's theory has (unfairly) had a bad press. For especially thin hulls, there is less reason to doubt the validity of the theory, and hence greater potential value in setting up such routines.

Most efforts on computing Michell's integral have been for mathematically-defined hulls, for which one, usually two, of the above integrals can be evaluated analytically. In particular, we have already mentioned and used the case of a parabolic strut, for which both z and x integrals are elementary, and even the final θ integral can be written in terms of Bessel functions. Although such idealised cases are useful to test the program, I waste no further time on them here and assume that all three integrals must be done numerically from input data for the actual offsets $Y(x, z)$.

In principle, all of the above numerical integration tasks are quite straightforward, and could even be done using standard (e.g. IMSL) packages. However, there are a number of special features, especially the singularity as $\theta \to \pi/2$. In that limit (which corresponds to the diverging part of the ship wave pattern), the effective wave number $\kappa \sec \theta$ becomes infinite, and the x integral has a highly-oscillatory integrand. This is a well-understood

problem, but it does require some care, especially in data specification near the bow and stern, which contribute most to the integrals in that limit. It is moderated, especially for submerged or semisubmerged hulls, by a decay factor from the z integral that (in spite of the $\sec^5 \theta$ factor in the final integrand), damps out contributions from near $\theta = \pi/2$.

The present implementation uses a special trapezoidal-like algorithm for the z integral, Filon's quadrature for the x integral, and Simpson's rule for the θ integral. The last needs little discussion. We divide the range $(0, \pi/2)$ into N_θ intervals, with N_θ typically 40, and use standard Simpson weights, calling up the integrand as a function of θ, via subroutines that evaluate the integrals for P and Q, described below.

Filon's quadrature (Filon 1929) is a procedure with the same accuracy as Simpson's rule that can account for rapidly-oscillating integrands. Specifically, it is exact whenever the function F appearing in the x integral is piecewise quadratic. If instead Simpson's rule was used for the x integral, this would be exact when the whole integrand (i.e. F times a cosine or sine) was piecewise quadratic. If there is no contribution from the extreme bow and stern (i.e. $Y = 0$ and hence $F = 0$ there), the algorithm used is simply

$$P(\theta) \approx \sum_{i=1}^{N_x-1} \omega_i F(x_i, \theta) \cos(\kappa x_i \sec \theta) \, \Delta x$$

where the length of the ship has been divided into N_x equal segments each of length Δx, i.e. there are $N_x - 1$ stations $x = x_i$, $i = 1, 2, ..., N_x - 1$, not counting the extreme bow $x = x_0$ or stern $x = x_{N_x}$. The even weights are

$$\omega_{2i} = (3K + K \cos 2K - 2 \sin 2K)/K^3$$

for all i, and the odd weights are

$$\omega_{2i+1} = 4(\sin K - K \cos K)/K^3$$

for all i, where $K = \kappa \sec \theta \Delta x$. When K is small, these weights tend to the standard Simpson values ($\omega_i = 2/3$ even, $4/3$ odd), so, as expected, the influence of the Filon method is felt mainly for large K, i.e. as $\theta \to \pi/2$.

The core of the implementation is the z integral. This has an integrand consisting of the input offset Y times an exponential decay factor (noting that z is always negative here). The latter causes little difficulty, and the ordinary trapezoidal or Simpson rule could have been used. However, there are potential advantages in an alternative Filon-like procedure (cf Tuck 1967) for which the answer is exact when Y itself, rather than Y times the exponential, is piecewise smooth. We assume it is piecewise linear, which gives trapezoidal-rule accuracy.

The actual algorithm used is

$$F(x, \theta) \approx \sum_{j=0}^{N_z} \omega_j Y(x, z_j) \exp(\kappa z_j \sec^2 \theta) \Delta z$$

where the section is divided in the vertical into N_z segments each of height Δz, with end-points z_j, $j = 0, 1, 2, ..., N_z$, and the weights ω_j are given by

$$
\begin{aligned}
\omega_0 &= \left(e^K - 1 - K\right)/K^2 \\
\omega_{N_z} &= \left(e^{-K} - 1 + K\right)/K^2
\end{aligned}
$$

for $j \neq 0, N_z$,

$$
\omega_j = \left(e^K + e^{-K} - 2\right)/K^2
$$

where

$$
K = \kappa \sec^2 \theta \Delta z
$$

(Note that this is a different K and different ω's from those for the above x-wise Filon quadrature). These weights reduce to those of the ordinary trapezoidal rule ($\omega_0 = \omega_{N_z} = 1/2$, $\omega_j = 1$ otherwise) when $K \to 0$, so the impact of the special algorithm is felt most for large K, i.e. again when $\theta \to \pi/2$. If such a provision is not made, unless the grid size Δz is very small, the correct rate of decay of the diverging wave contribution is not captured.

Although, since the z integral is independently evaluated for each separate value of x, we could choose a different grid for every station, it is convenient to use a universal set of waterplanes $z = z_j$ that is the same for all stations, and is specified in the input offset data. For semi-submerged hulls where the lowest point of the hull is higher near the bow and stern than at mid-section, this means that we must input and use in the above algorithm a certain number of artificial zero offsets. In that case $\Delta z = D/N_z$ where D is the draft at the deepest point of the hull, usually at midships.

Note that a feature of this implementation is that the z integral is exact for rectangular sectioned struts. Since the Filon rule is similarly exact for parabolic x-variation, the program yields identical output for parabolic struts to that of check routines that do the z and x integrals analytically.

5   Numerical Results

Various checks have been performed on this program, not only to verify its accuracy as a program for the Michell integral, but also to add to the store of information about the accuracy of that integral itself.

For example, the program self-checks when run for parabolic struts, where we can write down the exact values for the z and x integrals, and hence, in effect, exact values for the resistance. As indicated above, the nature of the numerical method being used is such that these exact answers are exactly reproduced by the program; hence in one sense there is no accuracy check here, only a check that there are no blunders in program or data preparation.

Some parabolic strut results were given earlier. In Figure 4 we repeat some computations of Chapman (1972) for a parabolic strut with $B/L = 0.15$ and $D/L = 0.44$. This

is thin compared to a ship in its cross-section shape ($B/D = .51$), but has a not particularly thin waterplane. The present computations agree with Chapman's, and confirm the remarkable agreement with Chapman's experiments.

Next, Figure 5 shows computations for an actual ship hull, namely the Taylor Standard Series hull of prismatic coefficient 0.56, with $L/B = 10.1$ and $B/D = 3$. Thus this hull is thinner in waterplane than the above strut, but it is not at all thin in section. This hull was studied by Graft et al (1964), and Figure 5 also shows their own computations and their experiments with both 6 metre and 3 metre models. Their computational method was a rather crude one of Havelock; nevertheless the absolute differences between the theories are comparable with the remarkably small differences between theory and experiment. Even more significantly, the latter are comparable to the (scale-effect) differences between the two sets of experiments. Note that the resistance is now scaled with respect to the wetted surface area A_W.

Naval architects pay much (too much?) attention to the low Froude number range, e.g. F below about 0.32 for conventional ships. In this range, where R is quite small (generally negligible compared to viscous resistance, noting that the Schoenherr friction coefflcient for a full-size ship is about 0.002), both theories seem to over-estimate R, the present theory a bit more than Graft et al, and the relative error is quite large, of the order of 100%. I believe this is nevertheless acceptable. Thin-ship theory is often said to exaggerate the humps and hollows in this range, and to over-estimate R, but this isn't happening here, and the correct theoretical wave resistance under-estimates the experimental measurements.

Indeed, I believe the Graft et al computation is not accurate in this range, and it is fortuitous that their computations are closer to the experiments in this range than the present computations. The special algorithms used in the present program are capable of following the relatively short waves that are important at low Froude number, whereas most of the older methods that use crude discretisations are inclined to exaggerate the bow-stern interference effect that starts to dominate when waves are short. The cause for residual discrepancies between theory and experiment is probably to be sought in quantities such as form drag. For example, there is a somewhat mysterious suggestion in the discussion by Kennell of Salvesen et al (1985) to add a fixed constant to the SWATH wave resistance coefficient to make it agree more closely with the measured "residuary resistance", and the authors in their reply suggest a value of 0.005 for this constant. I am skeptical about this, but it would certainly make some comparisons like those of Figure 5 look even better than they already do. Nevertheless, Figure 5 tends to suggest that it is in the range F above 0.33 (when the enormous increase in R starts) where we would have greatest confidence in the theory when applied to conventional ships.

Finally we turn to an actual SWATH. Salvesen et al provide computations for one called T-AGOS. It is unnecessary to go into detail about the shape of this body, which is well described in Salvesen et al. A set of input offset data $Y(x, z)$ was prepared, initially with $N_x = 20$ stations and $N_z = 20$ waterlines, and the program run with $N_\theta = 160$. Incidentally, the factor W^2 in the θ integral introduces more "wiggles" into the integrand for catamarans than for mono-hulls, thus demanding a larger value of N_θ.

The results are shown in Figure 6, together with the computations from the SWATH-GEN program and the experiments of Numata, both reported in Salvesen et al (1985). Both theories agree well with the experiments, the present a little better. Certainly there is adequate evidence that a straightforward application of thin-ship theory is at least as good for SWATHs as the somewhat more complicated procedure advocated by Salvesen et al.

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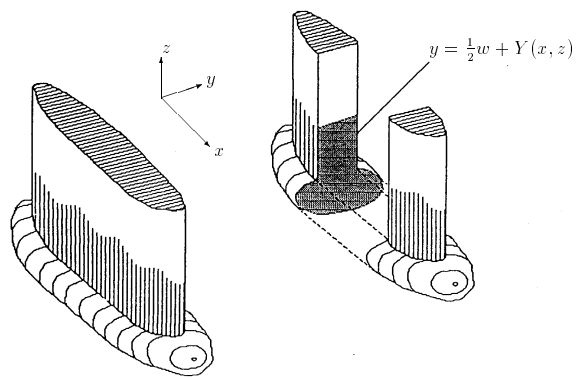Vossers, G. (1962). *Some applications of the slender body theory in ship hydrodynamics*. PhD thesis, Delft.

Figure 1: Sketch of a SWATH ship
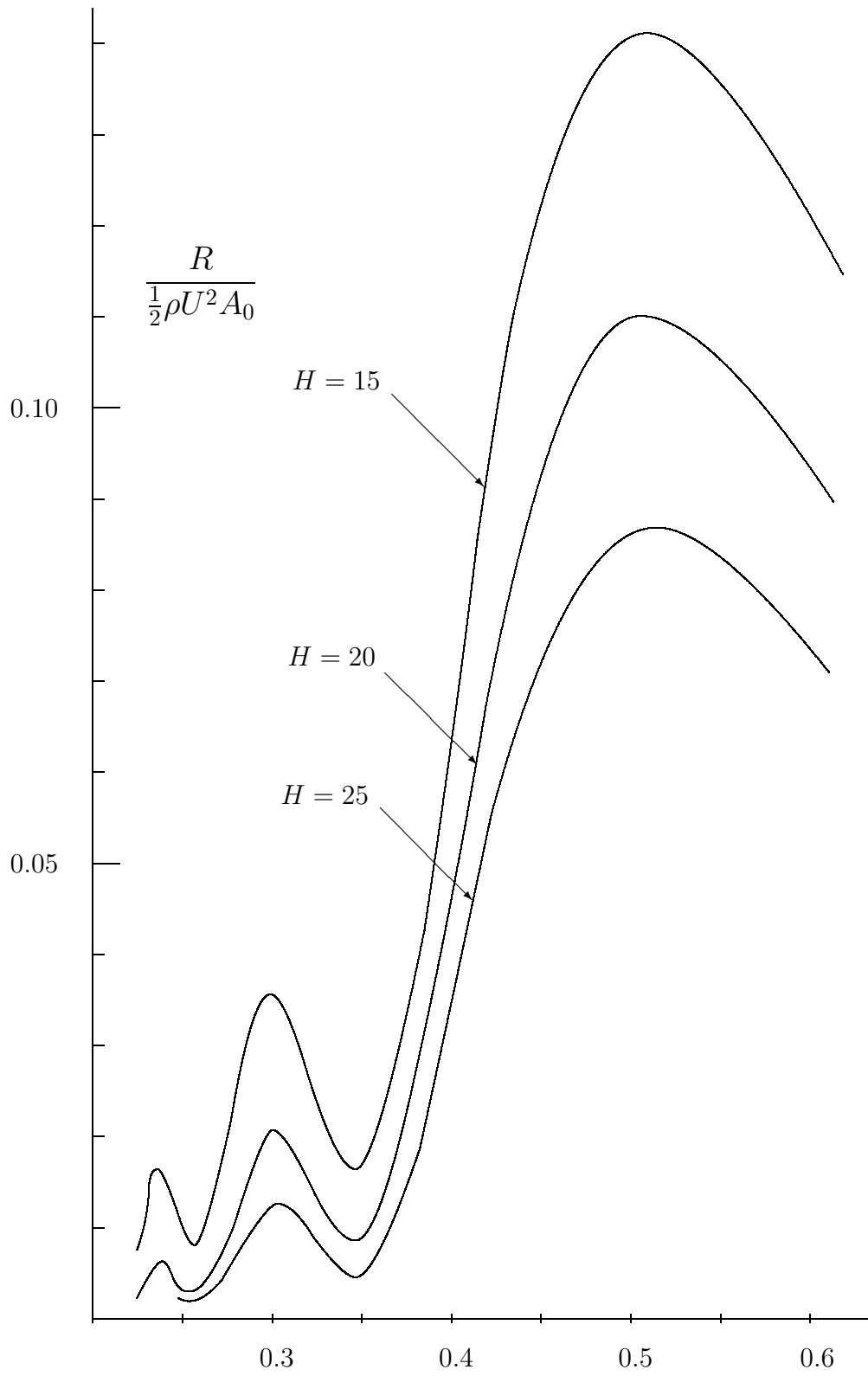
$$y = \tfrac{1}{2}w + Y(x,z)$$

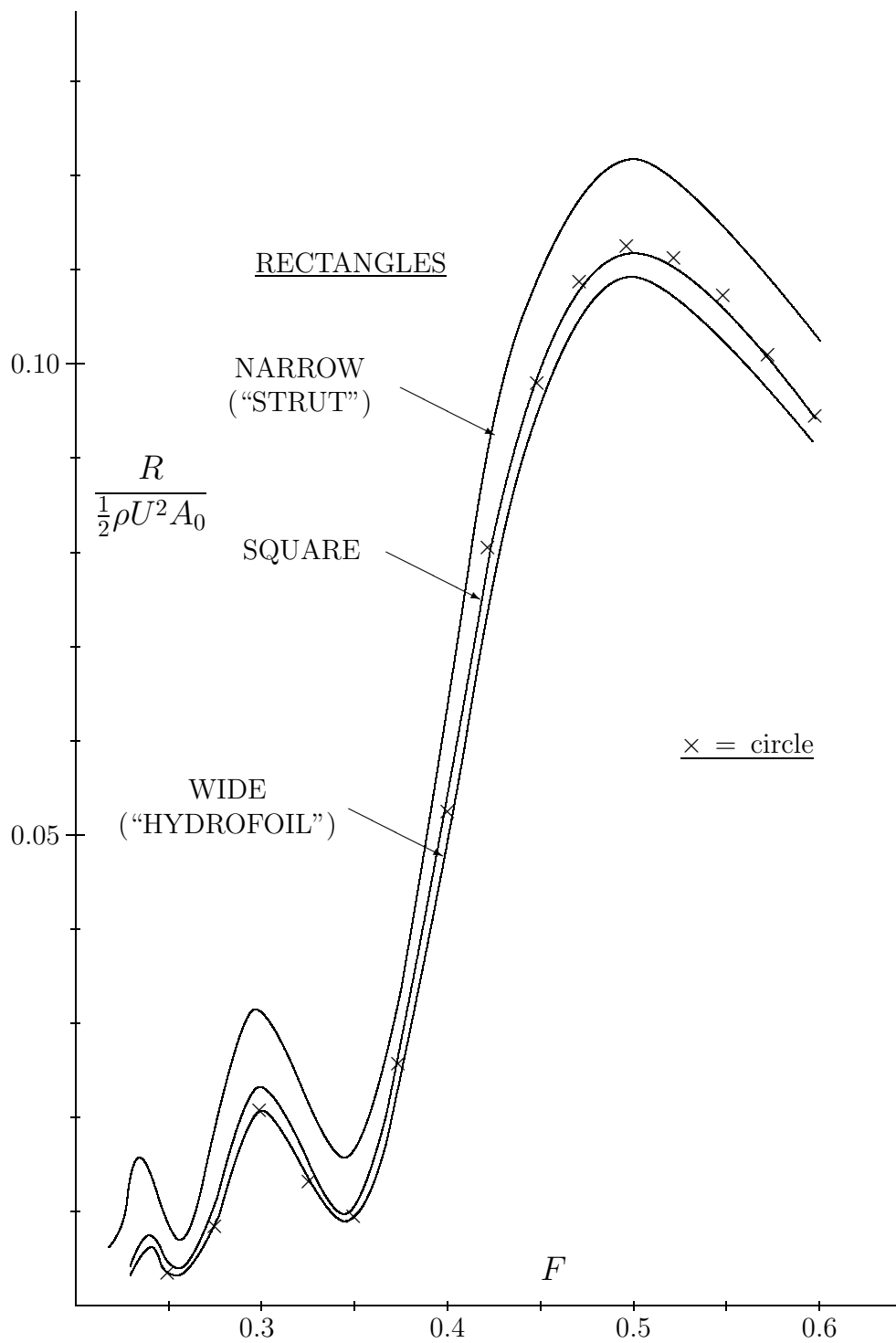Figure 2: Wave resistance of submerged slender bodies at various depths.

Figure 3: Wave resistance of submerged thin bodies of various cross sections.
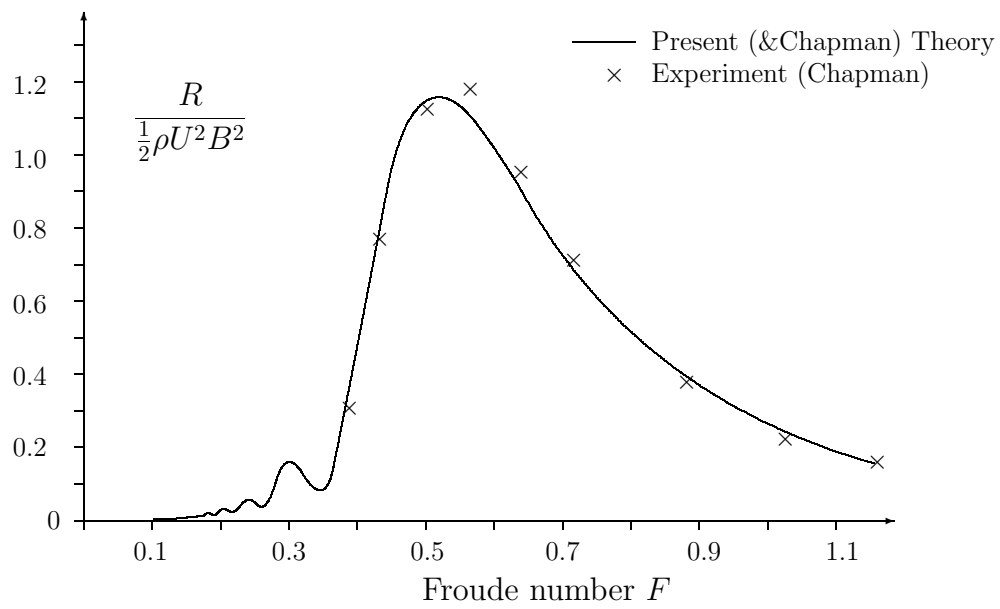
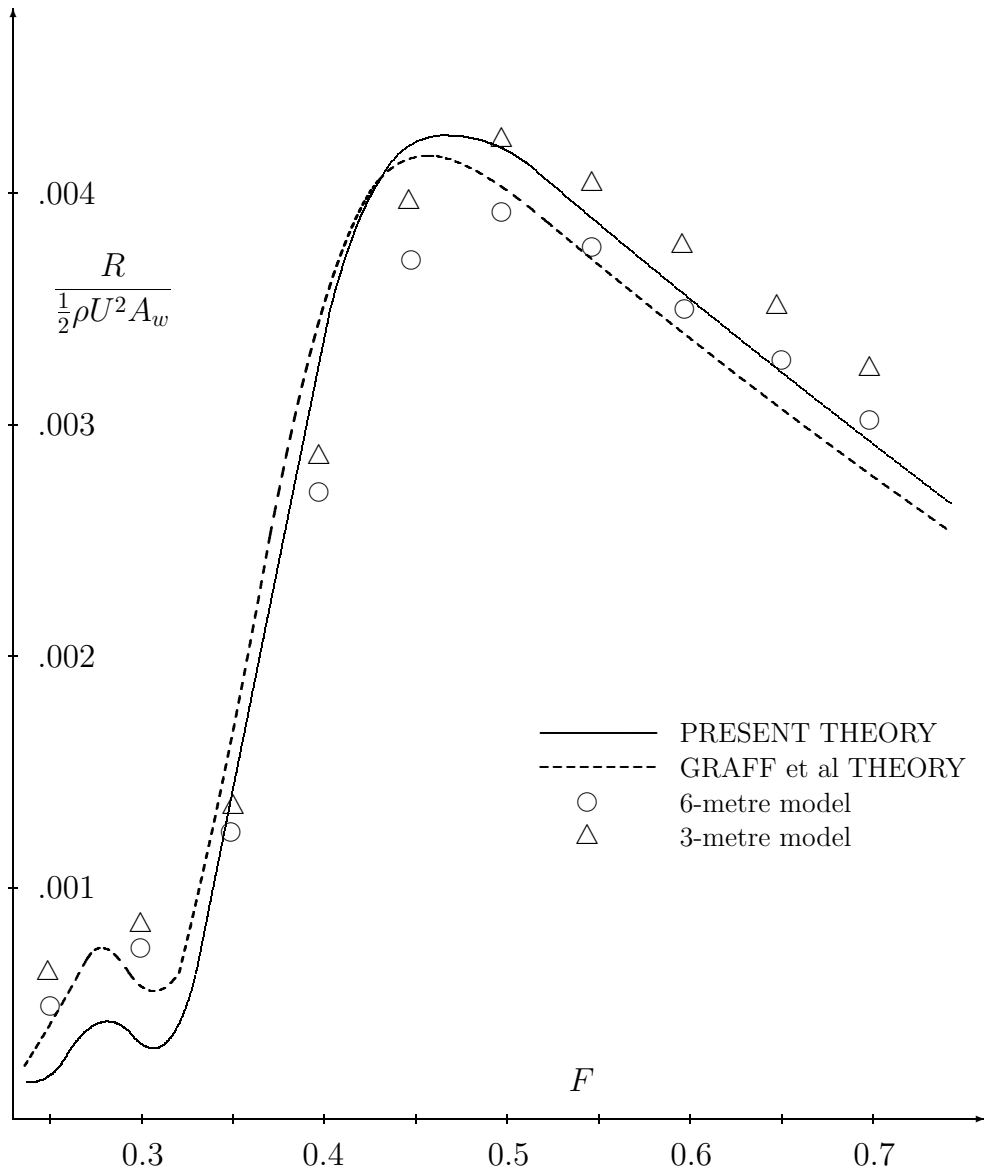Figure 4: Wave resistance of surface-piercing strut, compared to results of Chapman 1972.

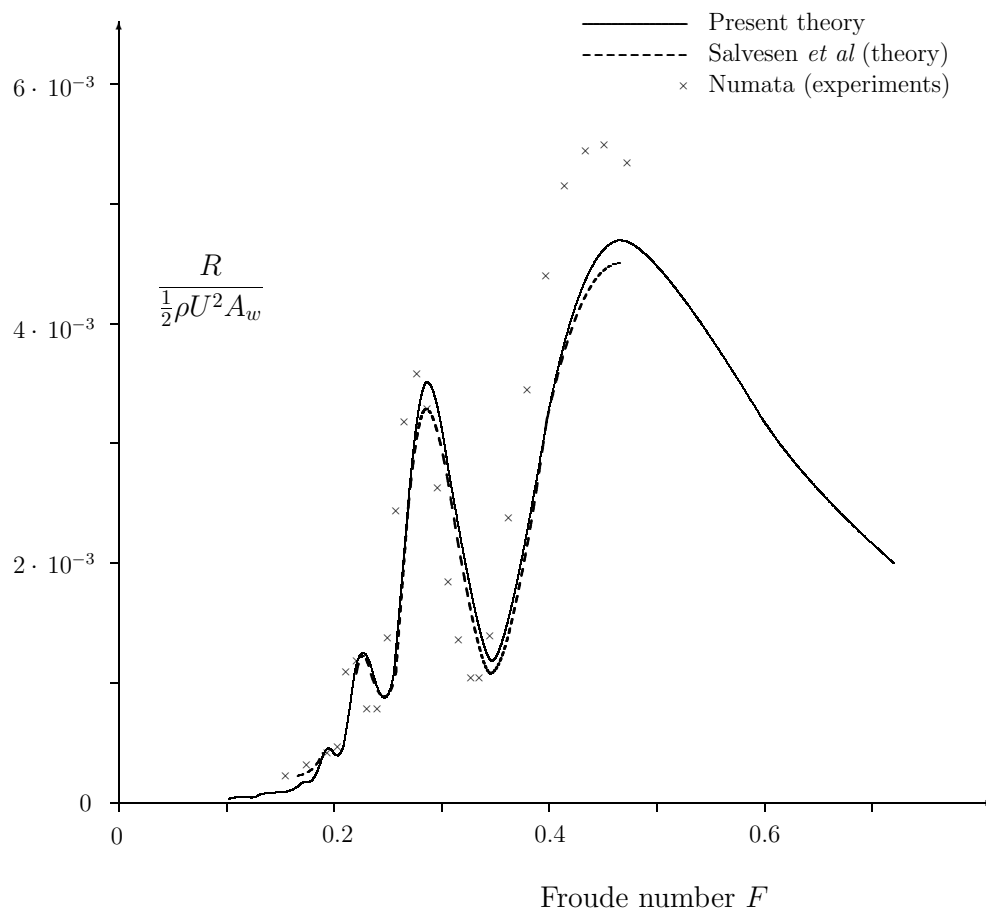Figure 5: Wave resistance of a Taylor standard series ship, compared to results of Graff et al (1964).

Figure 6: Wave resistance of the SWATH T-AGOS, compared to results of Salvesen et al (1985).