A NONLINEAR MATHEMATICAL MODEL OF MOTIONS OF A PLANING MONOHULL IN HEAD SEAS

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SUMMARY

A nonlinear mathematical model for the simulation of motions and accelerations of planing monohulls, having a constant deadrise angle, in head waves has been formulated. The model is based on 2-dimensional strip theory. The original model, developed by Keuning [9], who based his model on Zarnick [25], is extended to three degrees of freedom: surge, heave and pitch motion can be simulated. The simulations can be carried out for a planing boat sailing in (ir)regular head seas, using either a constant forward speed or constant thrust. The hydromechanic coefficients in the equations of motion are determined by a combination of theoretical and empirical relationships. The sectional hydromechanic forces are determined by the theory of a wedge penetrating a water surface. The wave excitation in vertical direction is directly integrated in the expressions for the hydromechanic forces and is caused by the vertical orbital velocity in the wave and the geometrical properties of the wave, altering the total wetted length and the sectional wetted breadth and immersion. An overall frictional resistance has been estimated. A constant thrust force can be set as input. When simulating with a constant thrust, the surge motion is induced by the frictional resistance and the horizontal component of the hydrodynamic force (hull pressure resistance).

The total calm water resistance has been validated. Existing experimental data of two models, a conventional double chined planing monohull (DCH) and a modern axebow (Axehull), planing in calm water showed a fair agreement with the calculated drag. A large sensitivity of the hydromechanic coefficients on the computed results for the total resistance, vertical motions and accelerations was found as well.

NOMENCLATURE

β	Deadrise angle of cross section
ν	Dynamic viscosity of water
τ	Propulsor shaft's angle
$\theta, \dot{\theta}, \ddot{\theta}$	Pitch angle, velocity and acceleration
ξ, χ, ζ	Body fixed coordinate system
A	Cross sectional area
a	Reduction length for transom correction function
a_{bf}	Buoyancy force correction factor
a_{bm}	Buoyancy moment correction factor
a_{nondim}	Dimensionless reduction length for transom correction function
A_w	Total wetted area
b	Half breadth of cross section
$C_{D,c}$	Cross flow drag coefficient
C_f	Friction coefficient
C_m	Added mass coefficient
C_{pu}	Pile-up factor
C_{tr}	Transom correction function
C_v	Froude number over breadth $\left(\frac{V_s}{\sqrt{g \cdot B_m}} \right)$
D	Total frictional resistance force along the hull
F_θ	Total hydromechanic pitch moment
F_θ'	Total hydromechanic pitch moment minus terms associated with motion acceleration
f_b	Sectional buoyancy force
f_{cfd}	Sectional viscous lift associated with the cross flow drag of a calm water penetrating wedge
F_{dyn}	Total hydrodynamic force

f_{fm}	Sectional hydrodynamic lift associated with the change of fluid momentum
$F_{N\nabla}$	Froude number over displacement $\left(\frac{V_s}{\sqrt{g \cdot \nabla^{1/3}}} \right)$
F_{sta}	Total hydrostatic force
F_x	Total hydromechanic force in x-direction
F_x'	Total hydromechanic force in x-direction minus terms associated with motion acceleration
F_z	Total hydromechanic force in z-direction
F_z'	Total hydromechanic force in z-direction minus terms associated with motion acceleration
h	Immersion of cross section
I_a	Pitch moment of inertia of the total added mass
I_{yy}	Pitch moment of inertia
L_c	Wetted chine length
L_k	Wetted keel length
L_m	Mean wetted length
M	Mass of ship
M_a	Total added mass
m_a	Sectional added mass
Q_a	Total added mass moment
R_F	Frictional resistance of bare hull
R_n	Reynolds number $\left(\frac{V_s L}{\nu} \right)$
R_P	Hull pressure resistance
R_{SR}	Total spray rails resistance
R_S	Total spray resistance
R_{VP}	Viscous pressure resistance
R_V	Total viscous resistance
R_W	Total wavemaking resistance
T	Thrust force

V	Water entry velocity of penetrating wedge
V_s	Forward speed
W	Weight of ship
w	Vertical orbital velocity at the undisturbed water level
x,y,z	Earth fixed coordinate system
x_1	x-coordinate measured from stern
x_a	Moment arm of hydrodynamic lift force
x_b	Moment arm of hydrostatic lift force
$x_{CG},\dot{x}_{CG},\ddot{x}_{CG}$	Displacement, velocity and acceleration of CG in x direction relative to earth fixed axes system
x_d	Moment arm of frictional resistance force
x_s,y_s,z_s	Steady translating coordinate system
x_t	Moment arm of thrust force
$z_{CG},\dot{z}_{CG},\ddot{z}_{CG}$	Displacement, velocity and acceleration of CG in z direction relative to earth fixed axes system

1 INTRODUCTION

The behaviour of planing monohulls in waves has been a widely researched topic since more and more semi-planing and planing monohulls appeared after the Second World War. Typical planing monohulls are: patrol vessels, pilot-boats, rescue vessels, coast guard vessels and small navy vessels.

The pressure acting on planing vessels running in calm water is characterized by a hydrostatic and hydrodynamic part. Due to the high forward speed and trim of the vessel there is a relative velocity between the hull and water and a hydrodynamic pressure proportional to the square of this relative velocity is generated. At high forward speed a large part of the weight of the vessel is carried by the dynamic pressure. In waves the relative velocity and thus the dynamic pressure gets additional contributions from the vessel's motions and the motions of the waves. The resulting nonlinear impact loads have a significant influence on the motions and accelerations in more or less every wave encounter and are crucial for the extreme responses.

For example, when such vessels are sailing in rough head seas, violent motions and large vertical acceleration peaks occur. The hull is subjected to high impact loads and the crew experiences high transient vertical accelerations and in most cases the crew needs to lower speed in order to avoid damage to the hull.

A good understanding of the behaviour of fast vessels in waves is necessary in order to be able to develop planing vessels with large operability. Moreover, most of the afore mentioned type of planing monohulls must be able to operate in (extremely) rough weather.

In [18] Savitsky presented an analysis is made of available data on the seakeeping behaviour of planing hulls in order to define and categorize those hydrodynamic problems associated with various speeds of operation in a seaway. He distinguished different behaviour in the low speed range $(F_{N_{\nabla}} < 2)$ (semi-displacement), where the seakeeping characteristics are very similar to the displacement hull and the high speed range $(F_{N_{\nabla}} > 2)$, where the hydrodynamic lift forces are predominate and where high impact forces can occur.

Fridsma [2, 3] executed systematic model tests with a serie of constant-deadrise models, varying in length. His results, presented in the form of response characteristics, cover a wide range of operating conditions and show, quantitatively, the importance of design parameters on the rough water performance of planing hulls. At this time it already became apparent that planing monohulls show a significant nonlinear behaviour in head waves.

The study of planing monohulls is closely related to the study of flat and V-bottom prismatic planing surfaces and to the study of a 2-dimensional wedge penetrating a calm water surface. These studies were initially carried out in order to get a better understanding of the hydrodynamics involved with the landing of seaplanes (for over nearly hundred years these topics have been studied, but the works of Von Karman [22] and Wagner [23, 24] can still be seen as the most important contributions in this field), but later were also used to get more insight of planing of monohulls, see for example Savitsky's work [17].

Zarnick [25], [26] [1]) developed a nonlinear mathematical model of motions of a planing monohulls in head seas, where the solution is solved in the time domain. His model is based on 2-dimensional strip theory and the forces acting on a cross section are determined by the theory of a wedge penetrating a fluid surface. The instantaneous values of wetted length, trim and sinkage are taken into account using strip theory in the time domain. The co-efficients in the equations of motion are determined by a combination of theoretical and empirical relationships. His model showed remarkably good agreement with experimental data.

His work forms the theoretical basis for the simulations models developed by Akers (Powersea) [1] and Keuning (Fastship) [9].

Garme [6, 4] developed a similar time domain simulation model for the motions of a planing monohulls in head seas, but his model distinguishes from Zarnick's model, because he implemented pre-calculated cross section data, so that the hull geometry is better accounted for.

Later, Garme [5] improved his time domain simulation model by adding a near-transom pressure correction function, which reduces the pressure near the stern gradually to zero at the stern.

In the present research the original mathematical model of motions of a planing monohull in head seas, developed by Zarnick and later extended by Keuning, is extended to three degrees of freedom: the surge, heave and pitch motion in (ir)regular head seas can be simulated. The simulations can be carried with either a constant forward speed or constant thrust.

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[1]This publication was not found, but is mentioned for the completeness

Fridsma [3] discovered in his extended research on the behaviour of hard-chine planing monohulls in head seas that little or no surge motion was measured for models sailing at high speeds. This would mean that if towed at constant forward speed, a model planing hull would behave exactly as if it would be tested at constant thrust. He proved that this hypothesis is also true for the lower speed range ($F_{N\nabla} \approx 1.5$). However, he only used one model and two seastates to proof his hypothesis. With the present computational model Fridsma's hypothesis can be verified or rejected, using the calculated results of more than two scenarios.

In the industry there is an increasing need to predict the motions and accelerations of a planing vessel in the design state. The nonlinear mathematical model developed in this research paper provides a computational design tool, with a rather simple input of the hull and little computer calculation time, for designers of fast planing monohulls to predict the operability in various sea states.

Moreover, in the near future, the effect of active control of the thrust (variation of the forward speed) when sailing in head seas on the vertical peak accelerations needs to be investigated. This mathematical model will be a valuable simulation tool for this research.

2 THEORETICAL BACKGROUND

The nonlinear mathematical model presented in this section is an extension of the work of Zarnick [25] and Keuning [9]. The simulation model is termed Fastship.

2.1 APPLIED THEORY, ASSUMPTIONS AND LIMITATIONS

Strip theory is used for the determination of the motions and accelerations of the system of a fast monohull in waves. When strip theory is used the assumptions are made that interaction effects within the 2-dimensional flows of the cross sections are negligible and thus that the hydromechanic forces, acting on the hull, can be approximated by integrating forces on cross sections over the ship's length.

Zarnick used the theory of a calm water penetrating wedge for determining the forces acting on a cross section. When looking at one slice of water (no waves), a planing monohull passing through it is like a wedge penetrating the water surface with a constant velocity, see figure 1.
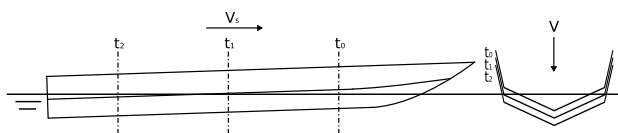

Figure 1: A planing monohull can be seen as a wedge penetrating the water surface

Since the theory of a penetrating wedge is used, hard chined hulls can be analysed with a better accuracy than

rounded bilges, since the model simplifies cross sections to a knuckled wedge.

Zarnick used the time domain approach for the determination of the behaviour of fast monohulls in head waves, because with the time domain approach the nonlinearities are seized better than with a frequency domain approach [12].

Furthermore, he assumed that the flow around the hull must be treated as quasi steady (every time instant the equilibrium of the forces and moments are analysed and from there the accelerations are determined) and that surface wave generation (wave resistance) and forces associated with unsteady circulatory flow can be neglected.

The wavelengths are assumed to be large in comparison with the ship's dimensions and the wave slope is small. Because of the large wavelengths diffraction forces can be neglected (only the Froude-Krylov forces are of importance).

The wave excitation in vertical direction is directly integrated in the expressions for hydromechanic forces and is caused by:

1. the geometrical properties of the wave, altering the total wetted length, the sectional wetted breadth and immersion and

2. the vertical orbital velocity.

Because the ships under consideration are generally shallow with respect to the height of the waves, the orbital velocity is taken at the undisturbed water surface in the plane $z = 0$.

The influence of the horizontal orbital velocities on both the horizontal and vertical motions is neglected, because these velocities are considered to be relatively small in comparison with the forward speed of the ship.

The wave excitation in horizontal direction is very difficult to model when applying strip theory. Together with the assumption that the wavelengths are large (small wave slopes), the present model is limited to moderate surge motions. Severe surge motions when diving into a wave when sailing in head seas cannot be simulated. The most important force in longitudinal direction at that specific moment is the wave excitation in horizontal direction (Froude-Krylov and diffraction).
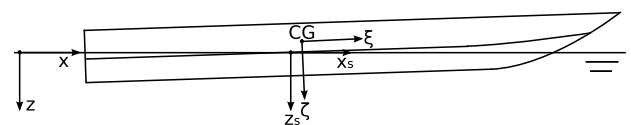
2.2 EQUATIONS OF MOTION

Figure 2: Coordinate system

The coordinate system used in the computational model is presented in figure 2. It consists of:

- an earth fixed coordinate system with x, y, z−axes, with the x−axis lying in the undisturbed water surface pointing in the direction of the forward velocity,

- a steady translating coordinate system with x_s, y_s, z_s axes, with the x−axis lying in the undisturbed water surface pointing in the direction of the forward velocity and travelling with a given constant velocity and

- a body fixed coordinate system with $\xi-$, $\chi-$ and $\zeta-$axes, with the origin in the centre of gravity of the ship and of which the $\xi-$axis is the longitudinal axis pointing forward.

The forces acting on a fast monohull are visualized in figure 3.
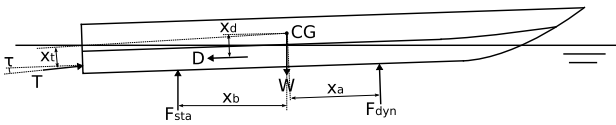
Figure 3: Definition of the forces acting on the ship

The equations of motion can be written as:

$$\begin{aligned}
M \cdot \ddot{x}_{CG} &= T\cos(\theta+\tau) - F_{dyn}\sin\theta - D\cos\theta \\
M \cdot \ddot{z}_{CG} &= -T\sin(\theta+\tau) - F_{dyn}\cos\theta - F_{sta} + D\sin\theta + W \\
I_{yy} \cdot \ddot{\theta} &= Tx_t + F_{dyn}x_a + F_{sta}x_b - Dx_d
\end{aligned}$$
$$(1)$$

The thrust force T is assumed to be constant. However, the efficiency loss of a (nearly) airborne propulsor has to be taken into account. In the computational model the total thrust efficiency decreases linear to zero with the submergence of the aft section.

In order to be able to investigate the effect of the surge motion on the vertical peak accelerations when performing simulations in head seas, a resistance dependent on the forward speed, wetted surface and pitch angle must be modelled. The estimate of the resistance will only be used to model a surge motion, not for resistance calculations. Therefore, constant and/or negligible small resistance components are left out of the equation of motion for surge. The inputted thrust might not be the actual thrust of the vessel, but may be somewhat smaller, due to the underestimation of the resistance.

Air friction is not taken into account. The superstructure is not defined in the computational model and the dependence of this resistance force on variation of the forward speed is assumed to be minimal.

According to Müller-Graf [13] the total bare hull resistance in calm water of a (semi-)planing monohull consist of the following components:

$$R_H = R_W + R_P + R_S + R_{SR} + R_V \qquad (2)$$

where:

- R_W: wave resistance

- R_P: hull pressure resistance (horizontal component of the dynamic lift force, here: $F_{dyn}\sin\theta$)

- R_S: spray resistance

- R_{SR}: spray rails resistance

- R_V: viscous resistance

Zarnick assumed that wave resistance can be neglected. However the wave resistance can be significant, especially when semi-planing. For now, this resistance component has been left out of the equation, especially because no direct formulation is available. The spray and spray resistance are difficult to model, although recently a paper has been published about this topic [19]. The results of that study have not yet been incorporated in the simulation model.

The viscous resistance of the bare hull consists of a frictional and a viscous pressure resistance component:

$$R_V = R_F + R_{VP} \qquad (3)$$

The viscous pressure resistance, caused by viscous effects of the hull shape and by flow separation and eddy making, can be neglected for $F_{N_\nabla} > 1.5$.

This leaves only two time dependent resistance components (see also [17]):

$$R_P = F_{dyn}\sin\theta$$
$$R_F = D$$

The determination of these resistance components will be explained in section 2.4 and 2.7.

2.3 SECTIONAL HYDROMECHANIC FORCES

The force acting on a cross section is visualized in figure 4 and consists of three components (force per unit length):

- a hydrodynamic lift associated with the change of fluid momentum (f_{fm})

- a viscous lift force associated with the cross flow drag of a water penetrating wedge (f_{cfd})

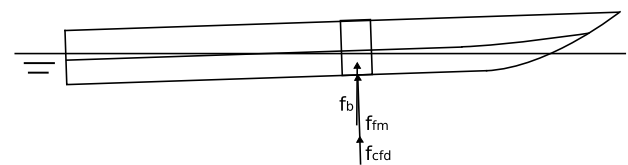- a buoyancy force related to the momentaneous displaced volume (f_b)

Figure 4: Orientation of forces acting on a cross section

The first force component prevails more in the fore part of the ship, where the chines are still dry, the last more in the

aft part, where the chines are immersed. The lift component associated with the cross flow drag of a penetrating wedge is small, but not negligible.

The sectional hydromechanic forces are determined according to the theory of a calm water penetrating wedge. The 2-dimensional penetrating wedge is replaced by a flat lamina by the assumption that the fluid accelerations are much larger than the gravitational acceleration [22, 23, 24]. The flat lamina is expanding at the same constant rate at which the intersection width of an immersing wedge is increasing in the undisturbed water surface, see figure 5. This expanding rate is dependent on deadrise angle:

$$\frac{db}{dt} = \frac{V}{\tan\beta} \tag{4}$$

Wagner included a term for water pile-up, which he gave the value of $\pi/2$.
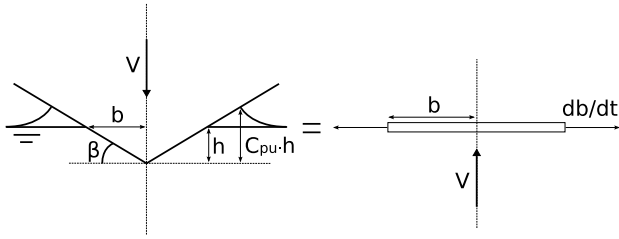
Figure 5: A wedge penetrating a calm water surface and expanding lamina theory

Payne [15] presented an approximation of the added mass variation with chines immersed and a conventional cross flow drag hypothesis as an additional lift component. He found, that the lift increment due to the chines immersed added mass variation is the same as the one due to the cross flow drag, so that adding the two together results in a chines immersed dynamic force which is twice the correct value.

In both Zarnick's as Keuning's computational model the additional lift component due to the cross flow drag has been applied.

Payne [14] also suggested that using a pile-up factor of $\pi/2$ too high impact loads were found when compared with experiments. Later, Payne [16] found that the results originally found by Pierson, in which he formulated that the pile-up is dependent on the deadrise, agreed very well with results found by Zhao and Faltinsen [27]. The expression for a deadrise dependent pile-up factor is:

$$C_{pu} = \frac{\pi}{2} - \beta\left(1 - \frac{2}{\pi}\right) \tag{5}$$

where a value for the pile-up factor of $\pi/2$ can be seen as the upper limit.

Hydrodynamic lift associated with the change of fluid momentum

The hydrodynamic lift associated with the change of fluid momentum is given by the rate of change of momentum of the oncoming fluid in terms of the added mass of the particular cross section under consideration:

$$f_{fm} = \frac{D}{Dt}(m_a \cdot V) = m_a \cdot \dot{V} + \dot{m}_a \cdot V - \frac{\partial}{\partial\xi}(m_a \cdot V) \cdot \frac{d\xi}{dt} \tag{6}$$

The difference with the ordinary strip theory methods is found in the time dependent added mass. Strip theory is 2-dimensional, therefore a lengthwise variation of the added mass has to be included, which is represented in the last term.

Change of the sectional added mass over the length plays an important role. Since the added mass of the sections is related to the beam of that section at the momentaneous waterline and since the beam of planing craft hulls generally decreases in the aft body to minimize wetted area, a negative lift could occur using these formulations. The formulation of the negative slope of the added mass is neglected if it occurs and the hydrodynamic lift force arising from the fluid momentum is set to zero for these sections [8].

The added mass for a penetrating wedge can be approximated by the high frequency solution:

$$m_a = C_m \cdot \frac{\pi}{2} \cdot \rho \cdot b^2 \tag{7}$$

and its time derivative as:

$$\frac{\partial m_a}{\partial t} = \dot{m}_a = C_m \cdot \pi \cdot \rho \cdot b \cdot \frac{db}{dt} \tag{8}$$

where C_m is the added mass coeffficient and $b = h \cdot \cot\beta$, in which h is the time dependent immersion of the wedge. When the term for pile-up is included, the breadth is expressed as: $b = C_{pu} \cdot h \cdot \cot\beta$. The determination of the added mass coefficient C_m will be explained in section 2.6.

Additional lift term due to cross flow drag

The additional lift term due the cross flow drag on the surface of a water penetrating wedge is expressed as:

$$f_{cfd} = C_{D,c} \cdot \cos\beta \cdot \rho \cdot b \cdot V^2 \tag{9}$$

where $C_{D,c}$ is the cross flow drag coefficient. The determination of the cross flow drag coefficient $C_{D,c}$ will be explained in section 2.6

Buoyancy force

The buoyancy force on a segment is assumed to act vertically and to be equal to the equivalent static buoyancy of the section multiplied with a correction factor a_{bf}:

$$f_b = a_{bf} \cdot \rho \cdot g \cdot A \tag{10}$$

where A is the immersed cross sectional area of the wedge.

The full amount of static buoyancy is never realized, because at the high speeds under consideration the flow separates from the chines and the stern, reducing the pressures at these locations to the atmospheric pressure. Therefore the total pressure distribution deviates considerably

from the hydrostatic pressure distribution when applying Archimedes Law. Therefore, the coefficient a_{bf} always has a value between 0 and 1. When the moment of this force is determined another correction factor, namely a_{bm}, is used.

The determination of the values of the buoyancy force and moment correction factors will be explained in section 2.6

2.4 TOTAL HYDROMECHANIC FORCES AND MOMENTS

The total hydromechanic force on the ship in the vertical plane is obtained by the summation of the three $(f_{ffm}, f_{cfd}$ and $f_b)$ force components for each segment and by integration of these sectional forces over the length of the ship , see figure 4.

The total hydromechanic force in each direction can be expressed as:

$$
\begin{aligned}
F_x &= -F_{dyn}\sin\theta = \\
&= -\int_L f_{ffm}\sin\theta d\xi - \int_L f_{cfd}\sin\theta d\xi = \\
&= -\int_L \left\{ m_a\dot{V} + \dot{m}_a V - U\frac{\partial}{\partial\xi}(m_a V) \right\}\sin\theta d\xi \\
&\quad - \int_L C_{D,c}\cos\beta \cdot \rho bV^2 \cdot \sin\theta d\xi
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
F_z &= -F_{dyn}\cos\theta - F_{sta} = \\
&= -\int_L f_{ffm}\cos\theta d\xi - \int_L f_{cfd}\cos\theta d\xi - \int_L f_b d\xi = \\
&= -\int_L \left\{ m_a\dot{V} + \dot{m}_a V - U\frac{\partial}{\partial\xi}(m_a V) \right\}\cos\theta d\xi \\
&\quad - \int_L C_{D,c}\cos\beta \cdot \rho bV^2 \cdot \cos\theta d\xi \\
&\quad - \int_L a_{bf} \cdot \rho gA d\xi
\end{aligned}
\tag{12}
$$

and the total hydromechanic pitch moment around the centre of gravity is expressed as:

$$
\begin{aligned}
F_\theta &= F_{dyn}x_a + F_{sta}x_b = \\
&= \int_L f_{ffm}\cdot\xi d\xi + \int_L f_{cfd}\cdot\xi d\xi + \int_L f_b\cos\theta\cdot\xi d\xi = \\
&= \int_L \left\{ m_a\dot{V} + \dot{m}_a V - U\frac{\partial}{\partial\xi}(m_a V) \right\}\cdot\xi d\xi \\
&\quad + \int_L C_{D,c}\cos\beta\cdot\rho bV^2\cdot\xi d\xi \\
&\quad + \int_L a_{bm}\cdot\rho gA\cdot\{\xi\cos\theta + \zeta\sin\theta\}d\xi
\end{aligned}
\tag{13}
$$

The velocities along U and normal V to the baseline of the ship can be expressed as:

$$
\begin{aligned}
U &= \dot{x}_{CG}\cdot\cos\theta - (\dot{z}_{CG}-w)\sin\theta \\
V &= \dot{x}_{CG}\cdot\sin\theta + (\dot{z}_{CG}-w)\cos\theta - \dot{\theta}\cdot\xi
\end{aligned}
\tag{14}
$$

And the acceleration normal to the baseline is expressed as:

$$
\begin{aligned}
\dot{V} &= \ddot{x}_{CG}\cdot\sin\theta + \ddot{z}_{CG}\cdot\cos\theta - \ddot{\theta}\cdot\xi \\
&\quad + \dot{\theta}\{\dot{x}_{CG}\cdot\cos\theta - \dot{z}_{CG}\cdot\sin\theta\} \\
&\quad - \dot{w}\cdot\cos\theta + \dot{\theta}\cdot w\cdot\sin\theta
\end{aligned}
\tag{15}
$$

The hydromechanic forces can now further be elaborated into:

$$
\begin{aligned}
F_x &= \left\{ -M_a\cdot\ddot{x}_{CG}\sin\theta - M_a\cdot\ddot{z}_{CG}\cos\theta + Q_a\cdot\ddot{\theta} \right. \\
&\quad -M_a\cdot\dot{\theta}(\dot{x}_{CG}\cos\theta - \dot{z}_{CG}\sin\theta) + \int_L m_a\dot{w}\cos\theta d\xi \\
&\quad -\int_L m_a\dot{\theta}w\sin\theta d\xi - \int_L \dot{m}_a V d\xi + \int_L UV\cdot\frac{\partial m_a}{\partial\xi}d\xi \\
&\quad -\int_L Um_a\frac{\partial w}{\partial\xi}\cos\theta d\xi - \int_L Um_a\dot{\theta}d\xi \\
&\quad \left. -\int_L C_{D,c}\cdot\cos\beta\cdot\rho bV^2 d\xi \right\}\sin\theta
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
F_z &= \left\{ -M_a\cdot\ddot{x}_{CG}\sin\theta - M_a\cdot\ddot{z}_{CG}\cos\theta + Q_a\cdot\ddot{\theta} \right. \\
&\quad -M_a\cdot\dot{\theta}(\dot{x}_{CG}\cos\theta - \dot{z}_{CG}\sin\theta) + \int_L m_a\dot{w}\cos\theta d\xi \\
&\quad -\int_L m_a\dot{\theta}w\sin\theta d\xi - \int_L \dot{m}_a V d\xi + \int_L UV\cdot\frac{\partial m_a}{\partial\xi}d\xi \\
&\quad -\int_L Um_a\frac{\partial w}{\partial\xi}\cos\theta d\xi - \int_L Um_a\dot{\theta}d\xi \\
&\quad \left. -\int_L C_{D,c}\cdot\cos\beta\cdot\rho bV^2 d\xi \right\}\cos\theta \\
&\quad -\int_L a_{bf}\cdot\rho gA d\xi
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
F_\theta &= Q_a\cdot\ddot{x}_{CG}\sin\theta + Q_a\cdot\ddot{z}_{CG}\cos\theta - I_a\cdot\ddot{\theta} \\
&\quad + Q_a\cdot\dot{\theta}(\dot{x}_{CG}\cos\theta - \dot{z}_{CG}\sin\theta) - \int_L m_a\dot{w}\cos\theta\cdot\xi d\xi \\
&\quad + \int_L m_a\dot{\theta}w\sin\theta\cdot\xi d\xi + \int_L \dot{m}_a V\cdot\xi d\xi \\
&\quad - \int_L UV\cdot\frac{\partial m_a}{\partial\xi}\cdot\xi d\xi + \int_L Um_a\frac{\partial w}{\partial\xi}\cos\theta\cdot\xi d\xi \\
&\quad + \int_L Um_a\dot{\theta}\cdot\xi d\xi + \int_L C_{D,c}\cdot\cos\beta\cdot\rho bV^2\cdot\xi d\xi \\
&\quad + \int_L a_{bm}\cdot\rho gA\cdot\{\xi\cos\theta + \zeta\sin\theta\}d\xi
\end{aligned}
\tag{18}
$$

where

$$
M_a = \int_L m_a\cdot d\xi
\tag{19}
$$

$$
Q_a = \int_L m_a\cdot\xi d\xi
\tag{20}
$$

$$I_a = \int_L m_a \cdot \xi^2 d\xi \tag{21}$$

2.5 NEAR-TRANSOM PRESSURE CORRECTION FUNCTION

Garme and Rosén [6, 7] studied the pressure distribution on the hull of planing craft in calm water, head and oblique regular and irregular waves. Later, Garme [5] formulated a correction operating on both the hydrostatic and the hydrodynamic terms of the load distribution. He based his correction on the assumption that the pressure is atmospheric at the dry transom stern. A strictly 2-dimensional analysis of the lift distribution on the planing hull over estimates the lift in near transom region [6], see figure 6.
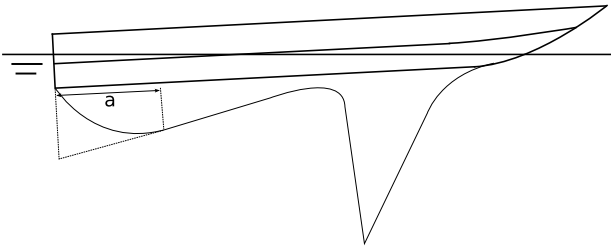
Figure 6: The principal lift distribution on a hull with a dry transom stern (the dotted line indicates the strictly 2-dimensional lift distribution)

Further, it is assumed that the difference between the 2-dimensional lift distribution and the actual pressure is largest at aft and decreasing afore. The correction approach is to multiply the 2-dimensional load distribution by a reduction function that is 0 at transom, that approaches 1 at a distance afore, and has a large gradient at aft which decreases towards zero with with increasing distance from the stern.

The near-transom pressure correction function is expressed as:

$$C_{tr}(x_1) = \tanh\left(\frac{2.5}{a} \cdot x_1\right) \tag{22}$$

in which a is a reduction length, see figure 6.

Garme rewrote the reduction length into nondimensional form:

$$a_{nondim} = \frac{a}{B_m \cdot C_v} \tag{23}$$

in which B_m is the full breadth of the mainsection and C_v is the Froudenumber over breadth: $C_v = \frac{V_s}{\sqrt{g \cdot B_m}}$.

After a systematic research on several model experiments Garme chose a value of 0.34 for a_{nondim}, which is applicable for medium and high speed configurations, $C_v \geq 2$.

The near-transom pressure correction function can now be rewritten into:

$$C_{tr} = \tanh\left(\frac{2.5}{0.34 \cdot B_m \cdot C_v} \cdot (\xi - \xi_{tr})\right) \tag{24}$$

in which ξ_{tr} is the body fixed coordinate of the stern. Garme validated the reduction function on basis of the model test measurements of the near-transom pressure, and on published model data on running attitude. This correction improves the simulation in both calm water and in waves for a wider speed range.

Although, a constant correction length is questionable if the ship motions are large and the wetted hull length is small as for sequences when the hull leaves or is close to leaving the water.

The transom reduction function reduces the sectional forces in the aft ship and has to be inserted within the integrals for the sectional hydrodynamic forces as follows:

$$F_x = -\int_L C_{tr} \cdot f_{fm} \sin\theta d\xi - \int_L C_{tr} \cdot f_{cfd} \sin\theta d\xi \tag{25}$$

$$F_z = -\int_L C_{tr} \cdot f_{fm} \cos\theta d\xi - \int_L C_{tr} \cdot f_{cfd} \cos\theta d\xi$$
$$-\int_L C_{tr} \cdot f_b d\xi \tag{26}$$

$$F_\theta = \int_L C_{tr} \cdot f_{fm} \cdot \xi d\xi + \int_L C_{tr} \cdot f_{cfd} \cdot \xi d\xi$$
$$+\int_L C_{tr} \cdot f_b \cos\theta \cdot \xi d\xi \tag{27}$$

2.6 DETERMINATION OF HYDROMECHANIC COEFFICIENTS

The integrals for the total hydromechanic forces and moments can be evaluated when the four hydromechanic coefficients ($C_{D,c}$, C_m, a_{bf} and a_{bm}) are known.

The lift force due to the cross flow drag is of minor importance, when compared with mass flux and buoyancy, so fixing the value of the cross flow drag coefficient has only a marginal effect on the total lift. Both Zarnick as Keuning fixed the value of $C_{D,c}$, according to the approach of Shuford [20]. Zarnick assumed that $C_{D,c} = 1.0$ and and Keuning assumed that $C_{D,c} = 1.33$. The latter one is used in the present computational model.

Originally, Zarnick used constant values for C_m, a_{bf} and a_{bm}. He assumed that the added mass coefficient C_m was equal to 1 and that the buoyancy correction a_{bf} was equal to $1/2$ and that a_{bm}, the correction for the longitudinal distribution of the hydrostatic lift, was equal to $1/2 \cdot a_{bf}$. He used a pile-up factor independent of deadrise: $C_{pu} = \pi/2$.

Keuning showed that Zarnick's method is only applicable to very high speeds, because of the constant values he used for the hydrodmechanic coefficients. Keuning, together with Kant [8], approximated the trim angle and sinkage of the craft under consideration using polynomial expressions derived from the results of systematic model tests, the Delft Systematic Deadrise Series (DSDS) [10, 11].

The solution of the equations of motion, describing the steady state planing in calm water, is known, because of these polynomial expressions. Substituting these values for sinkage and trim in the equations of motion results in a system of two equations and three unknowns. Keuning and Kant assumed that there is no additional factor for the correction of the longitudinal distribution of the hydrostatic lift: $a_{bm} = 1$. The values of C_m and a_{bf} can now be determined.

By determining the hydromechanic coefficients in this way, the hydrodynamic lift is brought into the computational model with a higher level of accuracy than in the original Zarnick model and the model can be used for a broader speed range. The present model is applicable for speeds ($F_{N_\nabla} > 1.5$), but it also restricted to hull forms similar to the models used in the DSDS.

Determination of hydromechanic coefficients C_m and a_{bf}

A planing vessel, sailing in calm water with a constant speed, is sailing in stationary condition. Sinkage and trim are constant in time. The sinkage and trim are determined by three components of the hydromechanic force in the vertical force and moment equilibrium. If only steady state planing is considered the following simplifications may be introduced in the equations:

$$\ddot{\theta} = \ddot{x}_{CG} = \ddot{z}_{CG} = 0$$
$$U = \dot{x}_{CG} \cdot \cos\theta \qquad (28)$$
$$V = \dot{x}_{CG} \cdot \sin\theta$$

The equations of motion in the stationary condition in calm water are reduced to ($\dot{x}_{CG}$ = constant):

Heave: $\qquad F_z^{'} + W = 0$

Pitch: $\qquad\quad F_\theta^{'} = 0$ $\qquad\qquad (29)$

where:

$$F_z^{'} = \int_L C_{tr}(\xi) \cdot UV \cdot \frac{\partial m_a}{\partial \xi} \cos\theta d\xi$$
$$\quad - \int_L C_{tr}(\xi) \cdot C_{D,c} \cos\beta \cdot \rho bV^2 \cos\theta d\xi \qquad (30)$$
$$\quad - \int_L C_{tr}(\xi) \cdot a_{bf} \cdot \rho gA d\xi$$

$$F_\theta^{'} = -\int_L C_{tr}(\xi) \cdot UV \cdot \frac{\partial m_a}{\partial \xi} \cdot \xi d\xi$$
$$\quad + \int_L C_{tr}(\xi) \cdot C_{D,c} \cos\beta \cdot \rho bV^2 \cdot \xi d\xi \qquad (31)$$
$$\quad + \int_L C_{tr}(\xi) \cdot a_{bf} \cdot \rho gA \cdot \{\xi\cos\theta + \zeta\sin\theta\} d\xi$$

in which $F_z^{'}$ and $F_\theta^{'}$ are the total hydromechanic forces minus terms associated with motion accelerations.

2.7 FRICTIONAL RESISTANCE FORCE

The total frictional resistance can be determined by:

$$D = C_F \cdot \frac{1}{2} \cdot \rho \cdot U^2 \cdot A_w \qquad (32)$$

The velocity along the baseline U can be expressed as: $U = \dot{x}_{CG} \cdot \cos\theta - \dot{z}_{CG} \cdot \sin\theta$. The influence of the orbital velocities is negligible and can therefore be omitted.

At each timestep the mean wetted length, the Reynolds number and the friction coefficient are calculated.

The mean wetted length is the average between the wetted keel length and wetted chine length and is formulated as:

$$L_m = \frac{L_k + L_c}{2} \qquad (33)$$

and the Reynolds number as:

$$R_n = \frac{U \cdot L_m}{\nu} \qquad (34)$$

The friction coefficient is determined using the ITTC formula.

$$C_f = \frac{0.075}{(\log R_n - 2)^2} \qquad (35)$$

The total wetted surface minus the dry stern is estimated by adding the surfaces of the wetted sections. The moment arm of this force is estimated by assuming that the centre of effort lies halfway the average immersion of the cross sections.

2.8 SOLUTION OF THE EQUATIONS OF MOTION

The equations of motion form a set of three coupled second order nonlinear differential equations, which are solved in the time domain using standard numerical techniques. The equations of motions can be written in matrix form:

$$M \cdot \ddot{\bar{x}} = \bar{F} \Rightarrow$$
$$\begin{pmatrix} M + M_a \sin^2\theta & M_a \sin\theta\cos\theta & -Q_a \sin\theta \\ M_a \sin\theta\cos\theta & M + M_a \cos^2\theta & -Q_a \cos\theta \\ -Q_a \sin\theta & -Q_a \cos\theta & (I + I_a) \end{pmatrix} \cdot \begin{pmatrix} \ddot{x}_{CG} \\ \ddot{z}_{CG} \\ \ddot{\theta} \end{pmatrix}$$

$$= \begin{pmatrix} T\cos(\theta+\tau) + F_x^{'} - D\cos\theta \\ -T\sin(\theta+\tau) + F_z^{'} + D\sin\theta + W \\ Tx_t + F_\theta^{'} - Dx_d \end{pmatrix}$$

$$\qquad\qquad (36)$$

in which $F_x^{'}$, $F_z^{'}$ and $F_\theta^{'}$ are the total hydromechanic forces minus terms associated with motion accelerations.

The solution of these sets may be found by:

$$\ddot{\bar{x}} = M^{-1} \cdot \bar{F} \qquad (37)$$

The procedure used to solve these equations is the Runge-Merson method. Knowing the vessel's orientation in the earth fixed coordinate system and the velocities at $t = t_0$ the equations are simultaneously solved for the small time increment dt to yield the solution on $t_0 + dt$.

3 VALIDATION OF CALM WATER RESISTANCE

Data of existing model tests were used to validate the total calm water resistance [21]. Two different hull shapes were used for these tests: a double chined planing monohull (DCH, 17° deadrise angle in aft ship) and a modern axebow (Axehull). The main dimensions are given in table 1 and a sketch of the hull geometries are given in figures 7 and 8.

Table 1: Main dimensions DCH and Axehull

Designation	Unit	DCH	Axehull
Length over all	m	19.34	20.00
Beam over all	m	6.3	5.65
Draft amidships	m	0.96	0.90
LCG rel to app	m	6.8	8.2
VCG	m	1.67	1.67
Displacement	m^3	33.66	35.22
k_{yy}	m	5.45	5.5
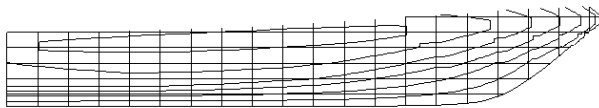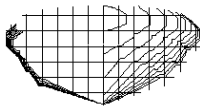
Figure 7: Sketch DCH
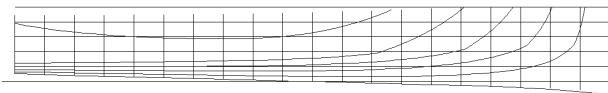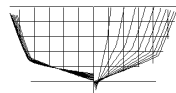
Figure 8: Sketch Axehull

The values of the hydromechanic coefficients a_{bf} and C_m were difficult to determine, because of the fact that the geometry of two models deviate significantly from the DSDS. Therefore the coefficients were estimated by using a parent hull from the DSDS with a comparable deadrise in the aftship (model 363, 19° deadrise angle) as a reference. This resulted in the following values:

Table 2: Used values of hydromechanic coefficients

F_{N_∇}	a_{bf}	C_m
$[-]$	$[-]$	$[-]$
<2.5	0.7	2
$2.5-3$	0.65	1.75
$3-3.5$	0.6	1.25
>3.5	0.5	1

For the validation the total resistance has been calculated at a constant forward speed. The working line of the thrust force T and the frictional resistance force D act through CG (no additional moments) and the vertical components of T and D are negligible small with respect to the other hydromechanic forces involved (no additional vertical force components) [25, 9].

Figures 9 to 13 show respectively the results of measured and calculated sinkage, trim, wetted surface, friction coefficient and resistance of the DCH.

The general trend of the sinkage has been captured, although a small deviation can be seen in the lower to middle speed range. At higher speeds the trim is underpredicted. Perhaps water spray on the two spray rails of the hull cause a larger trim angle than calculated. In the mathematical model no spray rails have been modelled. The wetted surface is underpredicted over the whole speed range. This results in a small underprediction of the total frictional resistance force, although a small overprediction of the friction coefficient compensates this effect slightly.

The residuary resistance is clearly underpredicted. The underprediction of the trim at higher speeds might yield a underestimation of the hull pressure resistance. At lower speeds the magnitude of the wave resistance might be significant. This resistance component should decrease towards higher speeds. At lower speeds, the calculated residuary resistance is about 25 percent of the measured residuary resistance, while at higher speeds it is 50 percent. The magnitude of the spray and spray rails resistance is still unknown.
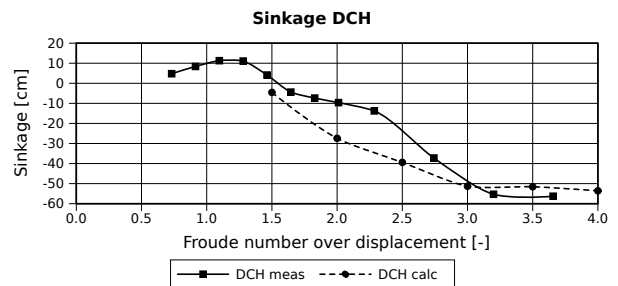

Figure 9: Measured and calculated sinkage DCH
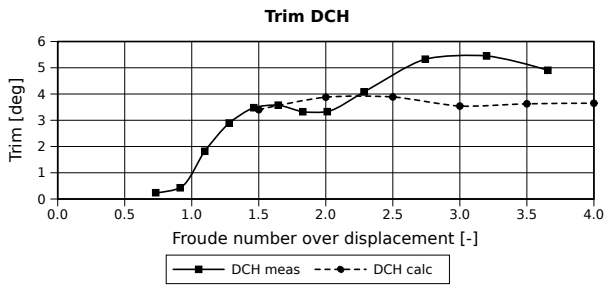
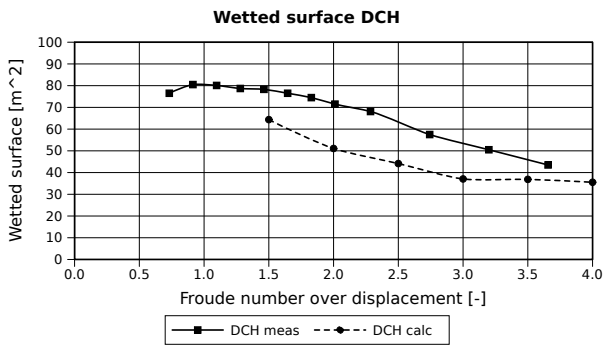Figure 10: Measured and calculated trim DCH

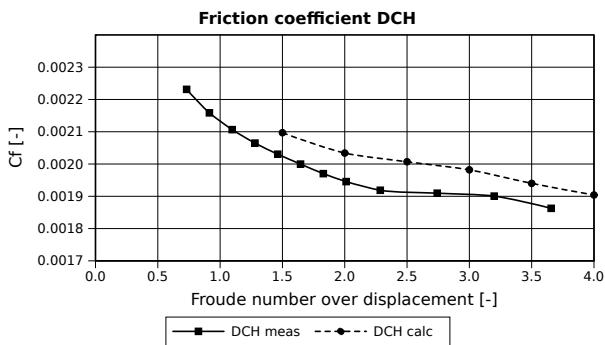Figure 11: Measured and calculated wetted surface DCH

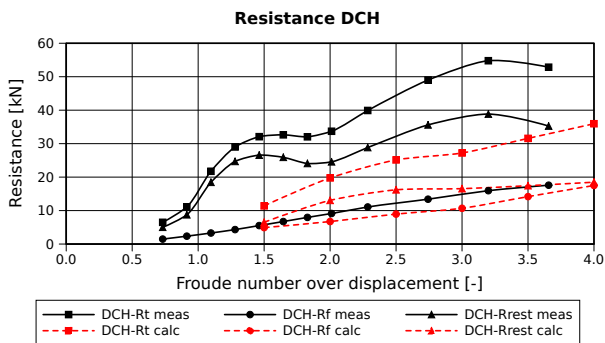Figure 12: Measured and calculated friction coefficient DCH

Figure 13: Measured and calculated resistance DCH

Figures 14 to 18 show respectively the results of measured

and calculated sinkage, trim, wetted surface, friction coefficient and resistance of the Axehull.
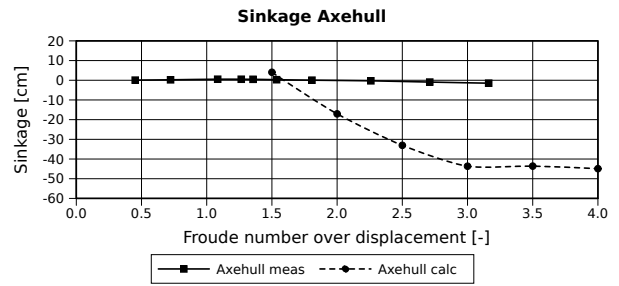

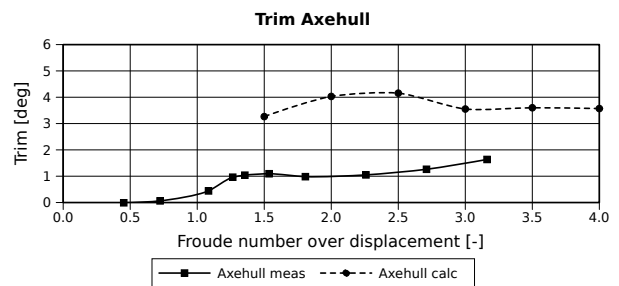Figure 14: Measured and calculated sinkage Axehull
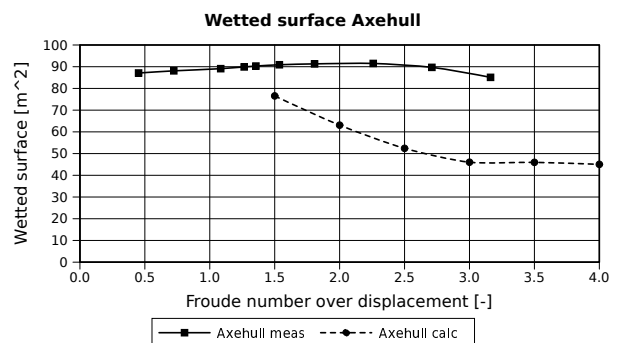

Figure 15: Measured and calculated trim Axehull

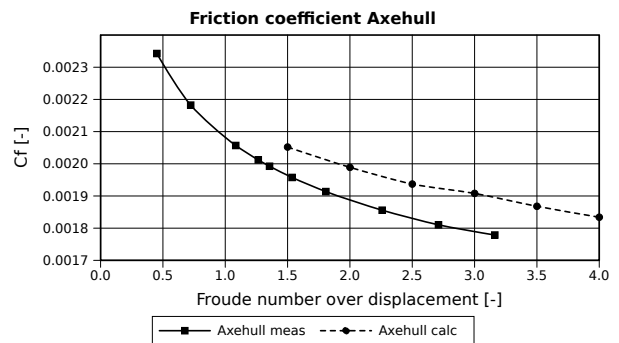Figure 16: Measured and calculated wetted surface Axehull

Figure 17: Measured and calculated friction coefficient Axehull
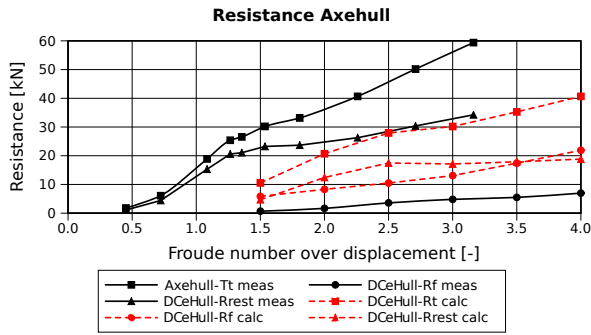
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Resistance Axehull

Figure 18: Measured and calculated resistance Axehull

The measured sinkage of the Axehull is nearly zero over the whole speed range. The mathematical model does not predict this constant value. The trim is overpredicted. The wetted surface is underpredicted over the whole speed range, probably because of the overprediction of the rise of the vessel. This results in an underprediction of the total frictional resistance force.

The erroneous calculated results for the sinkage and trim of the Axehull might also be caused by the estimated values for the hydromechanic coefficients. Perhaps the used values are not applicable for axebow hull shapes. At this moment it is not known what more appropriate values should be for these kind of hull shapes.

Generally, it can be concluded that the frictional resistance force is predicted accurate enough. But the residuary resistance and therefore the total resistance is clearly underpredicted using only the hull pressure resistance. The remaining resistance components should be incorporated into the model.

However, the mathematical is not developed for accurate calculations of the resistance; we are only interested in the time varying resultant force in longitudinal direction. For now, it is assumed that an accurate surge motion can be simulated, using only the hull pressure and frictional resistance.

4 SIMULATIONS ADDRESSING THE DIFFERENCE BETWEEN CONSTANT SPEED AND CONSTANT THRUST

Fridsma [3] observed little to no surge motion during his measurements at high speeds. Simulations with constant speed and thrust at high speeds show this trend as well. In the lower speed range he observed some surge motion. He tested a 10° deadrise model ($L/B = 5$, $B = 22.9$ cm) at $F_{N_\nabla} = 1.5$ in two sea states (a Pierson-Moskowitz spectrum with $H_s/B = 0.444$ and ($H_s/B = 0.667$), both with a constant thrust and constant speed. The average total resistance agreed very well. The distribution of the crest and throughs of the heave and pitch motions were nearly equal, as well as the distributions of the vertical accelerations at the bow and the centre of gravity.

Simulations carried out for the DCH and the Axehull in moderate sea states in order to address a possible difference between while simulating with a constant forward speed and a constant thrust, neither showed a remarkable difference. A wave realisation has been made according to the Jonswap spectrum. Three forward speeds ($V_s = 20, 30, 40$ kn), three significant wave heights ($H_s = 1.0, 1.5, 2.0$ m) and three peak periods ($T_p = 7, 10, 13$ s) have been chosen. The total run length was 1100 seconds. While simulating with constant thrust, the average forward speed over the total runlength has been used as a measure for the thrust.

The computational model has been validated for the motions in head seas by both Zarnick as Keuning using the results of model tests. They carried out model tests with constant forwards speed. Because of the fact that the results of simulations carried out with constant thrust show no remarkable difference with the results of simulations carried out with constant forward speed, it can be assumed that the motions and accelerations are predicted with the same level of accuracy as the original computational model.

However, the accuracy of the calculated results for the motions and accelerations for the Axehull is still questionable. The results are very sensitive to the used values of the hydromechanic coefficients.

5 CONCLUSIONS AND FUTURE WORK

A nonlinear mathematical model of a monohull having a constant deadrise angle, planing in head waves, has been formulated using strip theory. Keuning's [9] nonlinear mathematical model, based on Zarnick's model [25], has been extended to the possibilty to simulate with either constant forward speed or constant thrust.

The time domain approach is used for the determination of the motions. Each time step the sectional forces are elaborated and the total vertical and horizontal hydromechanic force and the total hydromechanic pitch moment are found by integrating the sectional forces and moments over the length of the vessel.

The surge motion is induced by a speed dependent frictional force and the horizontal component of the hydrodynamic force (hull pressure resistance), which varies with speed, trim angle and wetted surface. The thrust force is assumed to be constant. In order to find a more accurate surge motion, the wave, spray and spray rails resistance [19] still need to be incorporated.

Diffraction forces are neglected, only Froude-Krylov forces are of importance. Therefore the assumption is made that the wave lengths are long in comparison to the vessel's length and that wave slopes are small.

The coefficients in the equations of motion are determined by a combination of theoretical and empirical relationships. The two most relevant coefficients, the buoyancy correction factor and the added mass coefficient, are determined by the results of systematic model tests, the Delft Systematic Deadrise Series (DSDS). Polynomial expressions derived from the results of the DSDS approximate

the trim angle and sinkage for the situation of steady state planing in calm water. The values of the two most relevant coefficients are determined by substituting these values for sinkage and trim in the equations of motion.

However, this approach is very sensitive to errors. If the hull geometry deviates significantly from the DSDS, a value for the hydromechanic coefficients cannot be found and has to be estimated using a model of the serie with similar deadrise in the aftship.

In order to increase the level of accuracy of the computational model for modern hull shapes a thorough investigation into the hydromechanic coefficients should be carried out. Instead of applying a constant buoyancy correction factor and the added mass coefficient over the whole hull, a solution can be found in sectional hydromechanic coefficients, dependent on forward speed, deadrise, trim and sectional width. Research of the pressure distribution of planing V-bottom prismatic surfaces might give some insight in a finding a more accurate approximation of the (sectional) buoyancy correction factor and the added mass coefficient.

The computational model has been validated for the motions in head seas by both Zarnick as Keuning using the results of model tests. They carried out model tests with constant forwards speed.

Simulations with constant forward speed and constant thrust, carried out for a double chined hull (DCH) and an axebow hull shape (Axehull) showed no remarkable differences in motions and vertical accelerations. It can therefore be assumed that the motions and accelerations, calculated with constant thrust, are predicted with the same level of accuracy as the original computational model (constant forward speed). Fridma's hypothesis that model tests and thus simulations with constant forward speed generate the same results for the motions and vertical accelerations as model tests or simulations with constant thrust has been verified for moderate sea states.

However, it is still recommended to carry out model tests with free sailing self propelled models in head seas. The relation between thrust, resistance, motions, accelerations and wave profile needs to be studied more thoroughly. It is also expected that the number of (very) large vertical peak accelerations in higher sea states decreases when executing model tests with a constant thrust. Next, the influence of active control of the thrust can be studied using this simulation model.

The present nonlinear mathematical model of motions of a planing monohull in head seas provides designers of planing vessels a computational tool, with little calculation time, that is able to predict the surge, heave and pitch motion and the vertical accelerations in various seas states. The model is applicable for speeds larger than $F_{N_\nabla} \geq 1.5$. The geometry of cross sections of the hull are approximated by the shape of a hard chined wedge. The designer is able to analyse magnitude and probability of exceedence of large vertical peak accelerations when sailing in head seas.

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