A Statistical Analysis of FAO Resistance Data for Fishing Craft

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Analyse statistique des données FAO sur la résistance des carènes des bateaux de pêche

Analyse statistique détaillée des données rassemblées par la FAO sur les bateaux de pêche de nombreux pays. Les auteurs ont développé, en employant des méthodes statistiques d'analyse, une série générale d'équations exprimant le critère de résistance des carènes de ces bateaux pour une gamme étendue de nombres de Froude. Les résistances sont caractérisées par neuf paramètres de forme et de dimensions de coque pour une série de rapports vitesse/longueur, ce qui permet d'estimer les effets d'une variation des paramètres. En prenant pour base un bateau de pêche présentant des valeurs médianes des paramètres de forme, on a établi des graphiques montrant l'influence qu'exercent sur le critère de résistance de légères modifications apportées à chacun des paramètres de forme de coque. Il est possible d'obtenir des estimations de la résistance des carènes des bateaux de pêche en utilisant un calculateur électronique (KDF 9) avec un programme approprié. Par l'optimisation des équations présentées, il sera possible de créer de nouvelles formes de coque améliorées.

Análisis estadístico de los datos de la FAO sobre resistencia de las embarcaciones de pesca

El trabajo analiza detalladamente los datos de la FAO para las embarcaciones de pesca de muchos países. Se ha desarrollado una serie general de ecuaciones, empleando los métodos estadísticos de análisis para expresar los criterios de resistencia de estas embarcaciones con un amplio margen del número de Froude. La capacidad de resistencia ha sido expresada en términos de nueve parámetros de la forma y dimensiones del casco en una serie de razones velocidad-eslora y, por consiguiente, se pueden calcular los efectos de la variación de los parámetros. Se dan diagramas de una embarcación pesquera básica con valores medios de los parámetros de forma, mostrando los efectos en los criterios de resistencia de la realización de cambios discretos en cada parámetro de la forma del casco. Respecto a las embarcaciones de pesca, se puede actualmente calcular su capacidad de resistencia empleando una calculadora electrónica (KDF 9) y un programa para calculadoras preparado con este fin. Se sugiere el desarrollo de nuevas formas de casco mejoradas por optimización de las ecuaciones presentadas.

FAO has for some time collected results of model tests for fishing vessels in many countries (Traung, 1955 and 1959) and also sponsored a number of such tests. The published model tests have since been augmented by the results of further experiments from various additional sources and a complete list of the research organizations, foundation trusts, commercial firms and ship research laboratories who provided data is given at the end of this paper. FAO realized also that such results required systematic evaluation to make them of general value to all member governments, each with their varying fishing conditions and types of fishing craft.

Following an analysis of NPL data for trawlers made by Doust and Hayes (1962a, 1963), the first author was requested by FAO to prepare a report on the practicability of performing a similar analysis for smaller fishing vessels (Doust, 1962b). One of the main deterrents to systematic progress in fishing vessel designs below 100 ft in length and the determination of their resistance and propulsive qualities, has always been the relatively high cost of a model test, in relation to the capital cost of these smaller vessels. Another related consideration is that vessels in this category vary tremendously in main proportions and hull form characteristics, thereby making the cost of conducting any systematic experimental programme prohibitively large. The preliminary assessment of the data made by Tsuchiya (1964) according to the first author's recommendations, showed that large variations in resistance per ton of displacement were obtained between the best and worst forms included in the analysis. The excesses in resistance for constant displacement and Froude number ($v/\sqrt{gL}$) amount to between 70 and 100 per cent of the minimum recorded values over the range of speed-length ratio given by $V/\sqrt{L}=0.70$ to $V/\sqrt{L}=1.20$. ($F_n=v/\sqrt{gL}=0.209$ to 0.359). In this situation, as with the NPL data for larger deep-sea trawlers, the possibility of detecting the quantitative effects of individual hull form parameters on resistance becomes more attractive, whilst in addition there exists the strong possibility that relatively large gains in performance can be made relative to existing design practice. It should be recalled, moreover, that in some countries the hull dimensions or hull form parameters for fishing vessels can be restricted by local geographical or climatic conditions, and it is vitally important to have some means of assessing the penalties in performance thereby incurred, so that a complete technical and economic assessment of new fishing vessel requirements can be made.

TREATMENT OF INPUT DATA

Assessment of basic data: Before making a detailed statistical analysis of the data, it was necessary to decide upon the parameters of these forms which should be used to define their resistance performance. Previous experience with the NPL trawler data had shown that length/beam, beam/draft, maximum area coefficient, prismatic coefficient, longitudinal position of centre of buoyancy and the half-angle of entrance of the designed waterline were important parameters for such vessels. Other work for vessels of smaller length, tested at NPL, had shown the importance of afterbody shape in certain cases, particularly the penalties in performance incurred with high values of buttock slope. Before specifying the parameters used to define afterbody shape we note that as length between perpendiculars, commonly used in large vessels, became a rather meaningless dimension of length for the smaller vessels being considered, it was decided to use the FAO definition of absolute length on the floating waterline. This definition of length therefore includes that portion of the vessel incorporating the stern and avoids making separate distinctions between cruiser, transom and unorthodox sterns, although obviously artificial overhangs were faired out and an "equivalent" length determined. To cater for differences in afterbody shape then, two additional angles were evaluated for each design, viz., the maximum angle of run of any waterline up to and including the designed floating waterline ($\frac{1}{2}\alpha_r^\circ$) and the maximum buttock slope (α_{BS}°).

The maximum angle of run is measured at a section 5 per cent of the waterline length forward of the after end, whilst the maximum slope of the buttock line drawn at 25 per cent of the full beam is measured relative to the floating waterline. Although trim as such was not considered to be an important variable, its influence generally being reflected in a change of the remaining form parameters, it was decided to investigate its effect for these vessels as quite large variations in trim were apparent in the data. Trim is defined as the change in moulded draft at the forward and after ends of the floating waterline, expressed as a fraction of the length of the floating waterline. The following nine parameters of the hull shape and dimensions were therefore used to specify each vessel and evaluated up to the floating waterline:

viz., $[L/B, B/T, C_m, C_p, l.c.b.\%, \frac{1}{2}\alpha_e^\circ, \frac{1}{2}\alpha_r^\circ, \alpha_{BS}^\circ, \text{trim}]$
See nomenclature.

As in the earlier NPL analyses, the resistance performance criterion first proposed by Telfer (1933) was used, viz., $C_R = R{\cdot}L/\Delta V^2$ and values of this criterion were derived from the measured data for each model at discrete values of speed-length ratio. The values of speed-length ratio at which the data were scanned run from $V/\sqrt{L}=0.70$ to $V/\sqrt{L}=1.20$, at intervals of $V/\sqrt{L}=0.05$, making eleven values in all. We therefore aim to express the resistance criterion, C_R, as a function of the nine hull shape and dimension parameters, i.e.,

$$C_R = \psi[L/B, B/T, C_m, C_p, l.c.b.\%, \tfrac{1}{2}\alpha_e^\circ, \tfrac{1}{2}\alpha_r^\circ, \alpha_{BS}^\circ, \text{trim}] \quad (1)$$

in which Ψ will be estimated independently for each speed-length ratio considered. In order to make valid comparisons of performance and subsequent estimates of ship resistance, all the model data were standardized to a basic model length of 16 ft (4.877 m) using the 1957 ITTC formulation given by:

$$C_f = \frac{R_f}{\frac{1}{2}\rho S v^2} = 0.075\,[\log R_N - 2]^{-2} \quad . \quad (2)$$

Originally it had been considered that the analysis might be made retaining the Froude frictional coefficients, since the bulk of the FAO data sheets had been calculated on this basis. Subsequent re-examination however, showed that it was necessary to refer back to the basic model resistance-speed data in many cases, to achieve the required accuracy of resistance evaluation. It was therefore decided to use the basic model data throughout, applying a relatively small correction to the model results given by equation (2), in order to derive the equivalent resistance of a model having a waterline length of 16 ft (4.9 m). For subsequent extrapolation to full size, the ITTC or any alternative formulation can therefore be applied.

The resistance data for these vessels is derived from several sources, and inter-tank differences therefore had to be eliminated as far as possible, so that the real effects on resistance of parametric changes in hull shape and dimensions could be estimated. The following effects were considered likely to be present and included in the measured resistance data for each tank, and therefore should be quantitatively isolated, as far as possible, from the real effects being studied.

- Blockage effects on resistance due to changes in model size and tank dimensions
- Shallow water effects on resistance due to draft of the models in relation to tank depths
- Differences in measured resistance due to stimulated and unstimulated boundary-layer flows
- Differences in measured resistance due to model surface finish and different materials
- Differences in measured resistance due to dynamometer accuracy
- Effects on resistance of appendages fitted to some models
- Differences in measured resistance due to thermal gradients in the tank water
- Personal errors of observers recording the resistance and speed data

Blockage effects: The effects of tank blockage on the measured resistances of the models included in the analysis have been estimated using the type of correction proposed by Hughes (1961). This correction takes the form of a speed correction δv_m which when added to the speed of the model in the tank v_m, gives the speed of advance of the model in water of infinite breadth and depth, having the same resistance as the model in the tank. This correction is given by:

$$\left[\frac{\delta v_m}{v_m}\right]_b = \frac{\dfrac{\nabla_m}{l\cdot b\cdot h}}{\left[1 - \dfrac{\nabla_m}{l\cdot b\cdot h} - \dfrac{v_m^2}{gh}\right]} = B_1 \quad \text{say.} \quad . \quad (3)$$

Since the change in the model speed of advance δv_m is proportional to the slope of the resistance-speed curve, we have defined the slope of the $(C_R - V/\sqrt{L})$ curve as n at any point, in which case the change in C_R due to a corresponding change in $V/\sqrt{L}$ is given by:

$$C_{R_{(b)}} = \phi[B_1 n] \quad . \quad . \quad . \quad (4)$$

where ϕ is an auxiliary function to be determined from the subsequent analysis. Since Hughes' work suggests that

ϕ may be a simple linear function, we have included blockage terms in our expression for $C_{R_{16}}$ up to the second order, without much risk of losing any important effects, viz.,

$$C_{R_{(b)}} = a_r(B_1 n) + a_{r+1}(B_1 n)^2 \ldots \quad (5)$$

It should be noted, therefore, that the appropriate values of a_r and a_{r+1} will be determined by the subsequent statistical analysis, and it is only necessary to compute the appropriate values of "B_1" and "n" at each speed-length ratio being considered. This procedure not only reduces the magnitude of the already considerable analysis involved, but also avoids the use of the rather ill-defined values of a_r, a_{r+1} which would have to be applied in the range of speed-length ratio with which we are concerned for these vessels.

Shallow water effects: The exact solution, giving the correction to model speed due to the influence of shallow water on the resistance characteristics of a model being towed at speed v_m in a tank is given by Schuster (1955/56) as:

$$\left(1 + \frac{\delta v_m}{v_m}\right)_h^2 = \coth\left(\frac{gh}{(v_m + \delta v_m)^2}\right) \quad (6)$$

and this solution for $(\delta v_m/v_m)_h$ when $h \to \infty$ gives zero as one would expect. Hughes (1961) has given good approximate values of the function $[\delta v_m/v_m]_h$ for various values of (v_m^2/gh), so that the correction of each model results to allow for these shallow water effects can be readily obtained. Fortunately, this speed correction was found by Tsuchiya to be negligibly small for the FAO data and can therefore be ignored.

Turbulence stimulation of boundary-layer flow: In the previous NPL analyses for the large deep-sea trawlers, a correction allowing for laminar flow was applied to measured resistance values of models tested without fully-developed turbulent flow conditions. In this manner, the whole of the data was standardized to turbulent flow conditions, prior to subsequent analysis. Such corrections were relatively small and only for a few cases was the magnitude of the corrections up to 3 per cent of the measured resistance. In this case, for the FAO data, not only are there considerably more models involved, but the effects of turbulence stimulation are less well defined, as these vessels are rather outside the range of form parameters usually covered by studies of boundary-layer flow conditions. It was therefore decided to determine the effect of boundary-layer flow stimulation statistically, by including a term in the regression equation for C_R to estimate this effect. Since approximately half of the data were applicable to turbulent flow conditions and the other half unstimulated flow conditions, there was sufficient coverage of the data to make a first-order estimate of this effect at each speed-length ratio considered.

Effects on resistance of hull appendages: A study of the data showed that approximately 60 per cent of the models were tested with wooden keel pieces, which could be expected to produce an increase in resistance relative to naked models built to the moulded lines and, excluding the keel as an appendage. It was therefore necessary to allow for the differences in measured resistance, relative to a naked model, and an appropriate term was added to the regression equation for $C_{R_{16}}$ to estimate the influence of the wooden keel piece independently, at each speed-length ratio.

Effects of other factors on resistance: The effects on resistance measurement of model surface finish, materials used in their manufacture, dynamometry, thermal gradients and possible local currents induced in the tank water and errors of the personnel conducting the experiments cannot be quantitatively assessed without independent examination, and must be regarded in our analysis as random errors. The question arises as to the possible magnitude of their combined effects, in relation to the order of accuracy required in making realistic estimates of ship performance. An error of ± 1 per cent in speed estimation for the ship, considered to be sufficiently realistic for most practical requirements, allows a permissible variation in resistance estimation of between ± 5 per cent and ± 7 per cent in most cases with which we will be concerned for fishing vessels (resistance varies between $(\text{speed})^5$ and $(\text{speed})^7$). Our aim therefore is to formulate an equation for $C_{R_{16}}$ in terms of the nine hull parameters and the auxiliary functions expressing the effects of blockage, wooden keels and turbulent flow stimulation, such that the residual errors given by the differences between measured and estimated values for each model are of the order of 5–7 per cent or better, in each case.

THE HULL FORM PARAMETERS

Prior to the commencement of the computational work, it was necessary to study the dependency of the data values of each form parameter on those of all the others. In the ideal situation, each parameter will vary over its full range for the whole range of values of each of the other parameters and a rectangular distribution of data points will be revealed by plotting each pair of parameters on the usual Cartesian co-ordinates. Fig 1–36 show the distributions of data points for all the models used in this analysis, which are mainly derived from European or American tanks, and comprise a total of 308 designs. A larger amount of data is available from Japanese and other sources, but, since these cover an even wider range of parameter values, it was decided to make a separate study of these data at a later stage when further experience of the use of the results of this first analysis has been obtained. For the present data it was found that all

TABLE 1

Form parameter	*Extreme values*	*Range within which independence of parameters is applicable*
L/B	2.8 to 5.8	3.1 to 4.3
B/T	1.5 to 4.0	2.0 to 3.2
C_m	0.44 to 0.88	0.5 to 0.8
C_p	0.48 to 0.73	0.55 to 0.65
$l.c.b\%(-\text{ aft})$	−12.0 to +3.5	−6.0 to +1.0
$\frac{1}{2}\alpha_e^\circ$	6 to 40	15 to 34
$\frac{1}{2}\alpha_r^\circ$	22 to 80	30 to 60
α_{BS}°	10 to 57	16 to 34
trim (+ by stern)	−0.04 to +0.13	−0.04 to +0.13

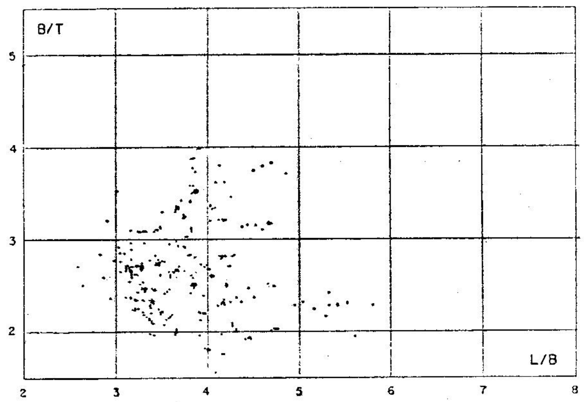


Fig 1. Relation between B/T and L/B of models analysed in the computer study

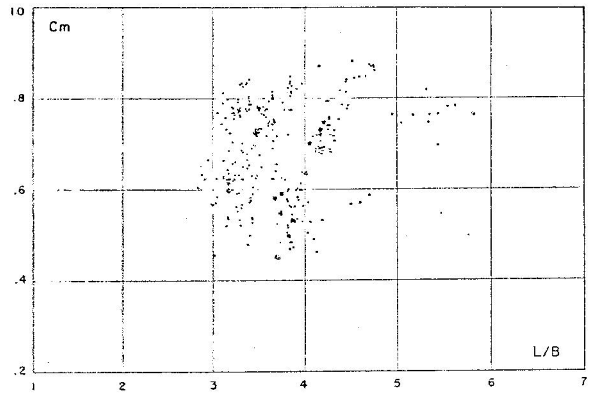


Fig 2. Relation between C_m and L/B of models analysed in the computer study

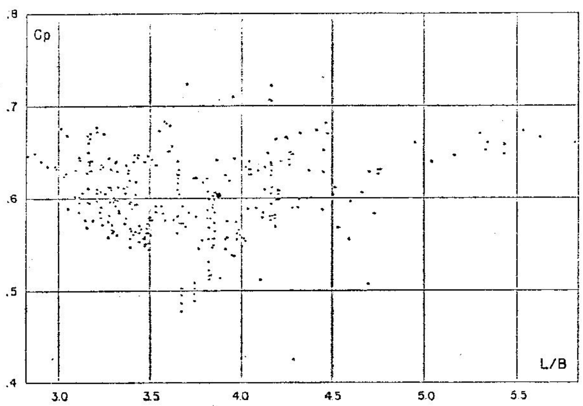


Fig 3. Relation between C_p and L/B of models analysed in the computer study

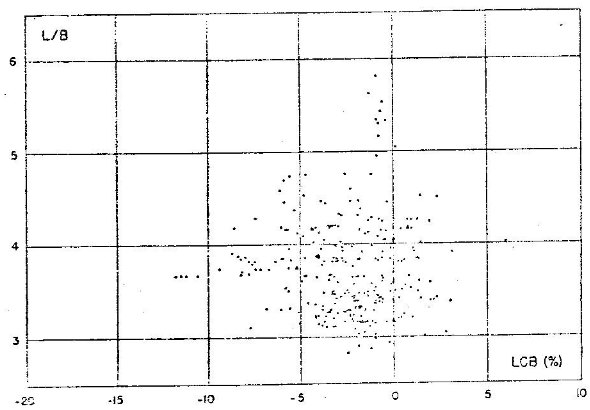


Fig 4. Relation between L/B and l.c.b.(%) of models analysed in the computer study

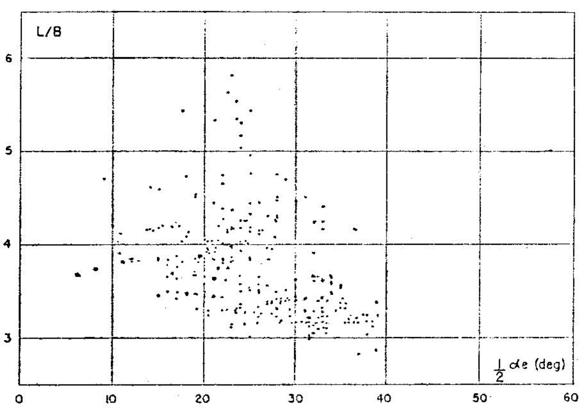


Fig 5. Relation between L/B and $\frac{1}{2}\alpha_e$(deg.) of models analysed in the computer study

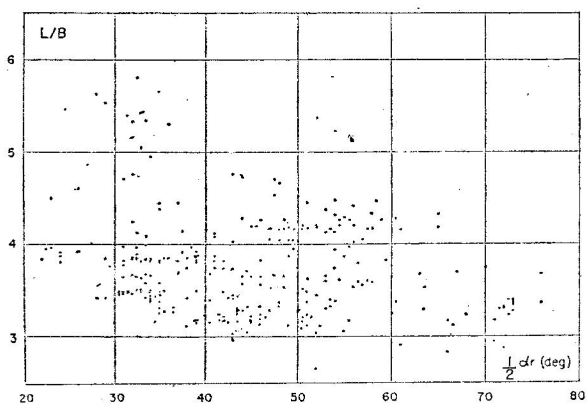


Fig. 6. Relation between L/B and $\frac{1}{2}\alpha_r$(deg.) of models analysed in the computer study

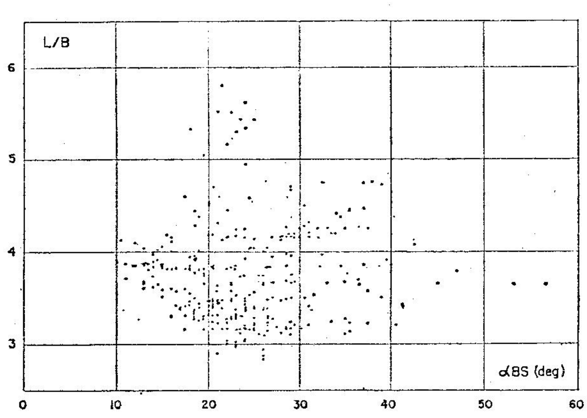


Fig 7. Relation between L/B and α_{BS}(deg.) of models analysed in the computer study

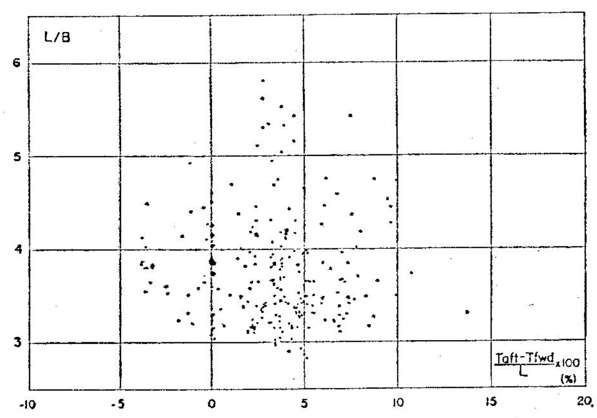


Fig 8. Relation between L/B and trim of models analysed in the computer study

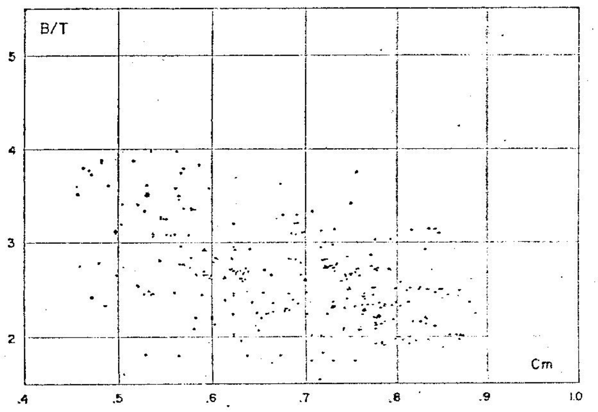


Fig 9. Relation between B/T and C_m of models analysed in the computer study

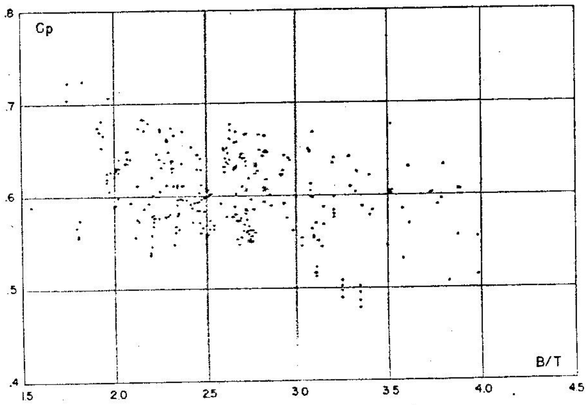


Fig 10. Relation between C_p and B/T of models analysed in the computer study

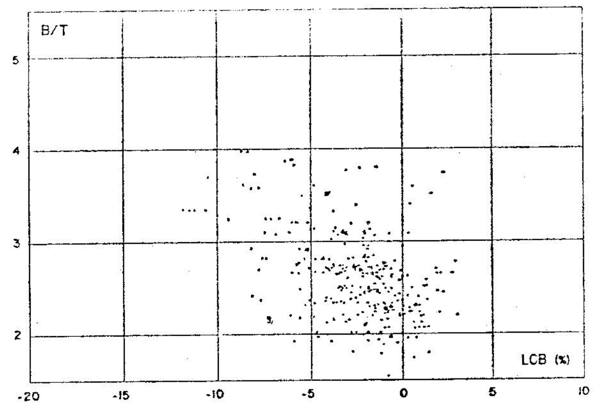


Fig 11. Relation between B/T and l.c.b.(%) of models analysed in the computer study

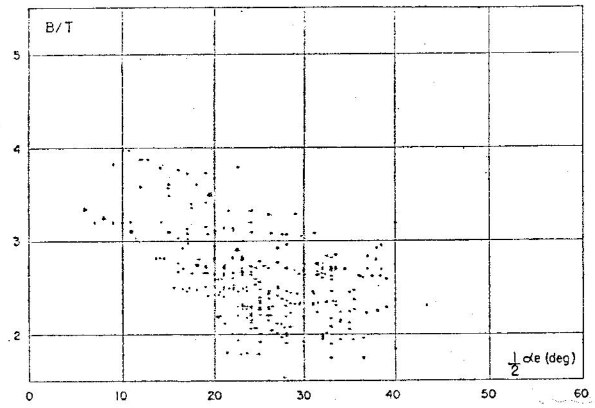


Fig 12. Relation between B/T and $\frac{1}{2}\alpha_e$(deg.) of models analysed in the computer study

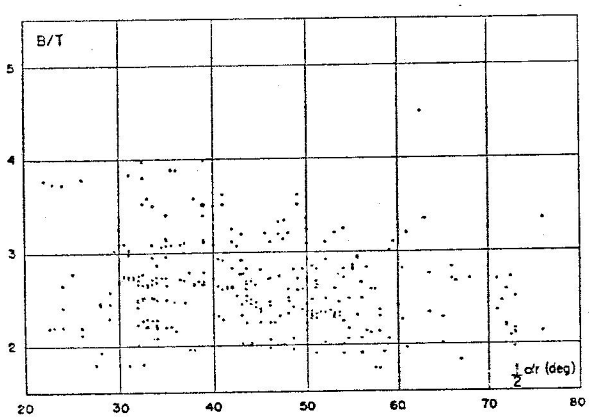


Fig 13. Relation between B/T and $\frac{1}{2}\alpha_r$(deg.) of models analysed in the computer study

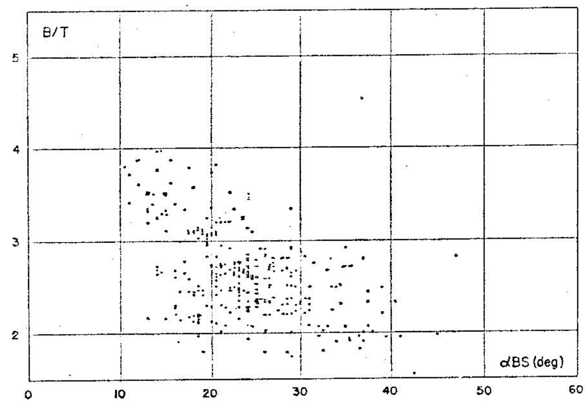


Fig 14. Relation between B/T and α_{BS}(deg.) of models analysed in the computer study

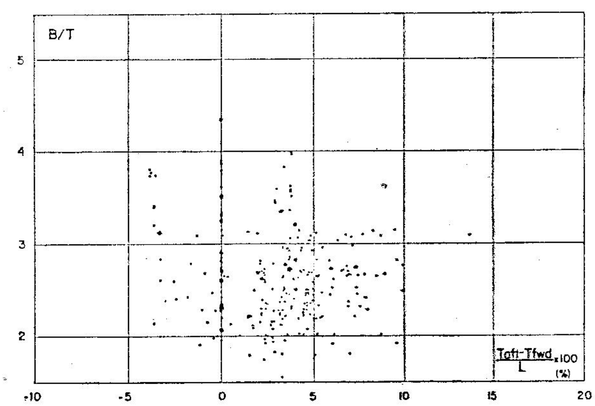


Fig 15. Relation between B/T and trim of models analysed in the computer study

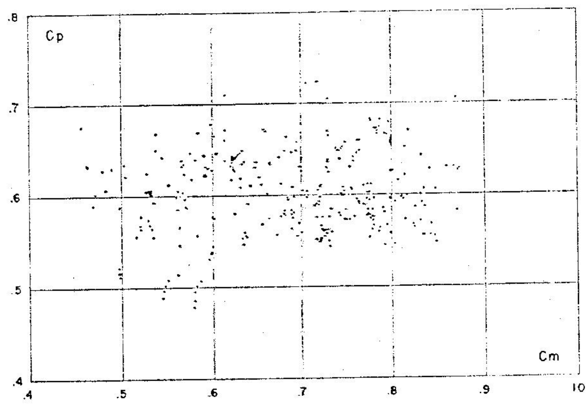


Fig 16. Relation between C_p and C_m of models analysed in the computer study

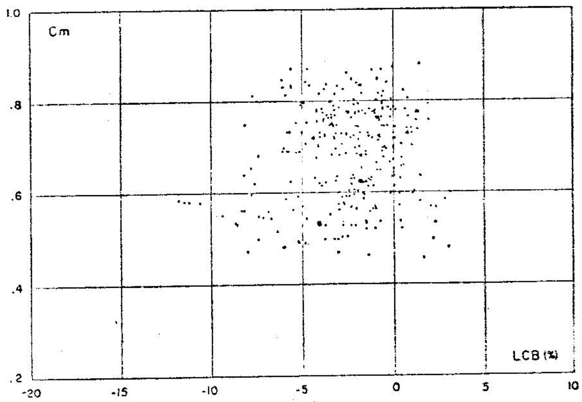


Fig 17. Relation between C_m and l.c.b.(%) of models analysed in the computer study

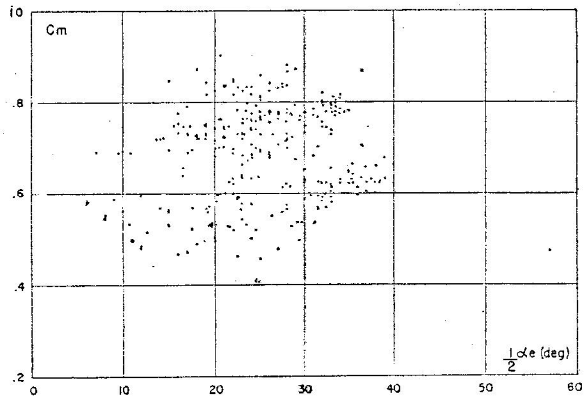


Fig 18. Relation between C_m and $\frac{1}{2}\alpha_e$(deg.) of models analysed in the computer study

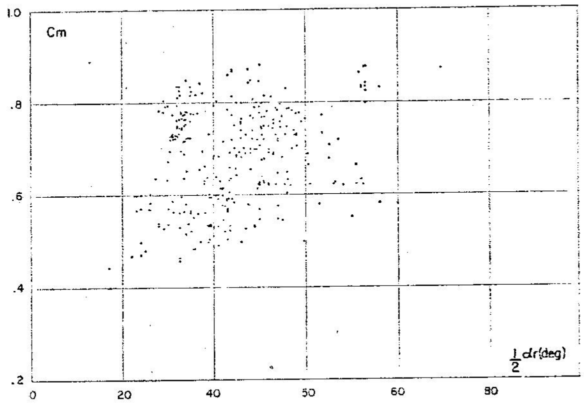


Fig 19. Relation between C_m and $\frac{1}{2}\alpha_r$(deg.) of models analysed in the computer study

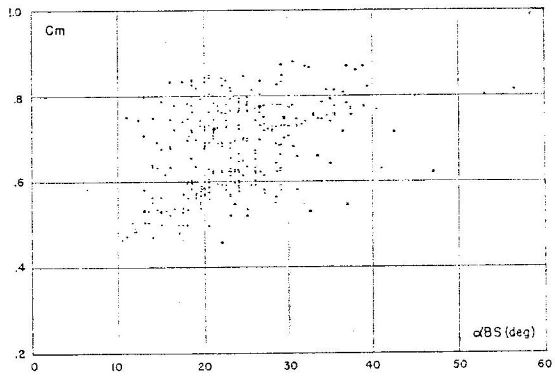


Fig 20. Relation between C_m and α_{BS}(deg.) of models analysed in the computer study

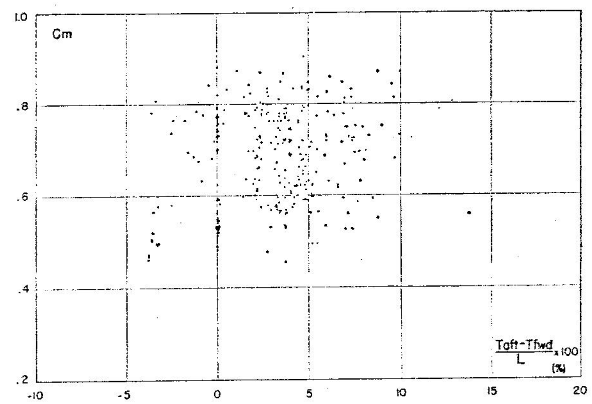


Fig 21. Relation between C_m and trim of models analysed in the computer study

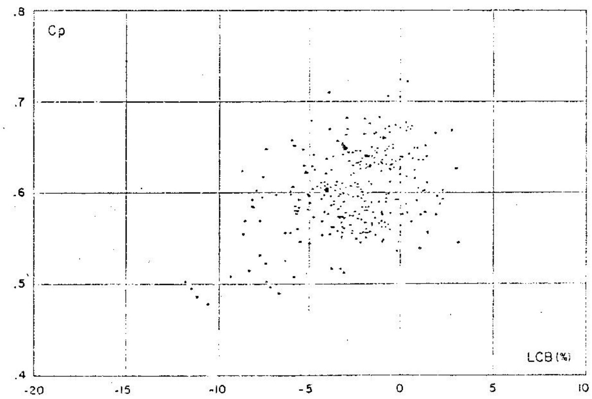


Fig 22. Relation between C_p and l.c.b.(%) of models analysed in the computer study

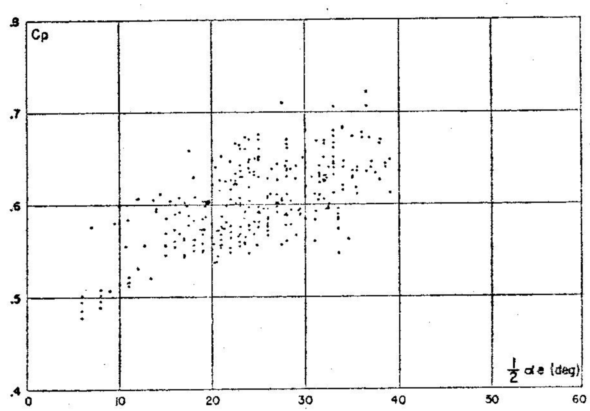


Fig 23. Relation between C_p and $\frac{1}{2}\alpha_e$(deg.) of models analysed in the computer study

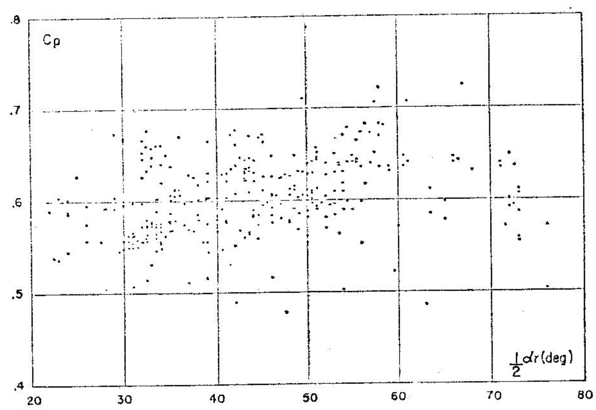


Fig 24. Relation between C_p and $\frac{1}{2}\alpha_r$(deg.) of models analysed in the computer study

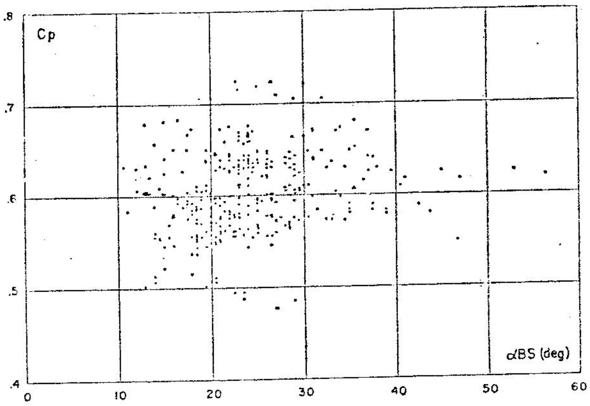


Fig 25. Relation between C_p and α_{BS}(deg.) of models analysed in the computer study

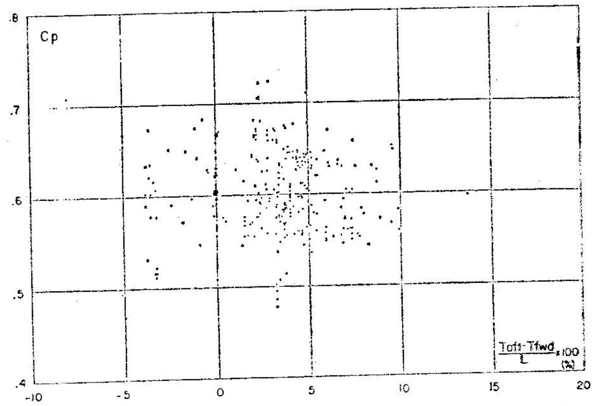


Fig 26. Relation between C_p and trim of models analysed in the computer study

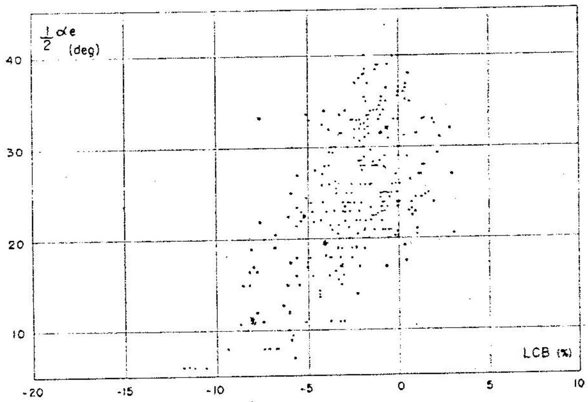


Fig 27. Relation between $\frac{1}{2}\alpha_e$(deg.) and l.c.b.(%) of models analysed in the computer study

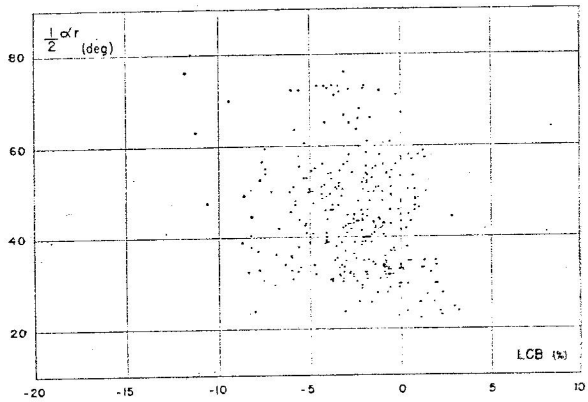


Fig 28. Relation between $\frac{1}{2}\alpha_r$(deg.) and l.c.b.(%) of models analysed in the computer study

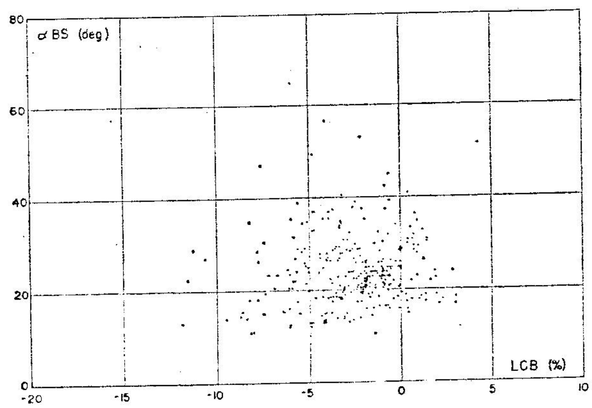


Fig 29. Relation between α_{BS}(deg.) and l.c.b.(%) of models analysed in the computer study

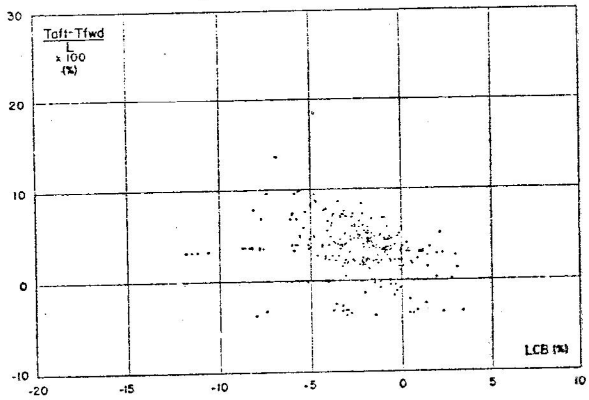


Fig 30. Relation between trim and l.c.b.(%) of models analysed in the computer study

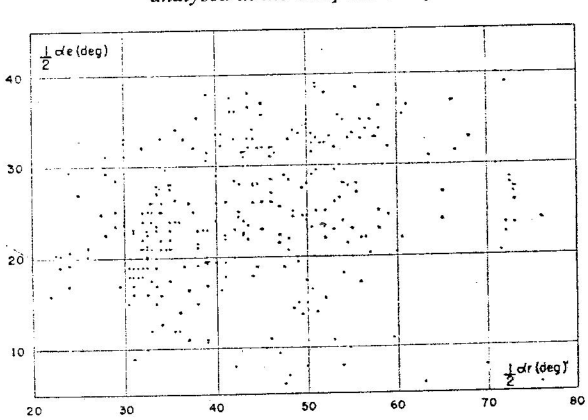


Fig 31. Relation between $\frac{1}{2}\alpha_e$(deg.) and $\frac{1}{2}\alpha_r$(deg.) of models analysed in the computer study

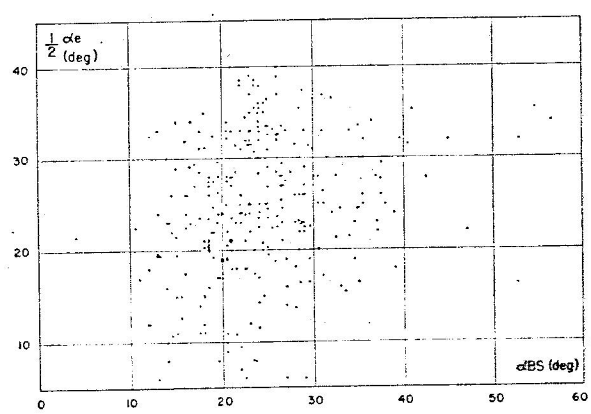


Fig 32. Relation between $\frac{1}{2}\alpha_e$(deg.) and α_{BS}(deg.) of models analysed in the computer study

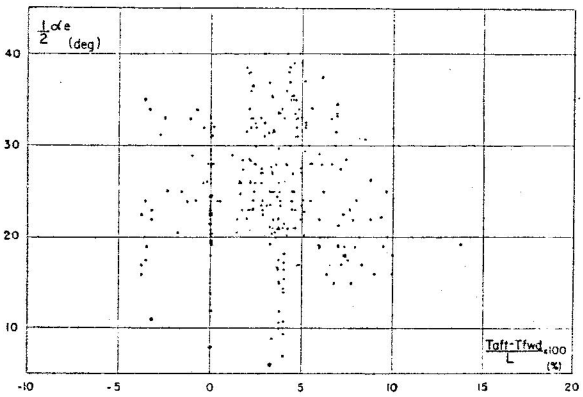


Fig 33. Relation between $\frac{1}{2}\alpha_e$(deg.) and trim of models analysed in the computer study

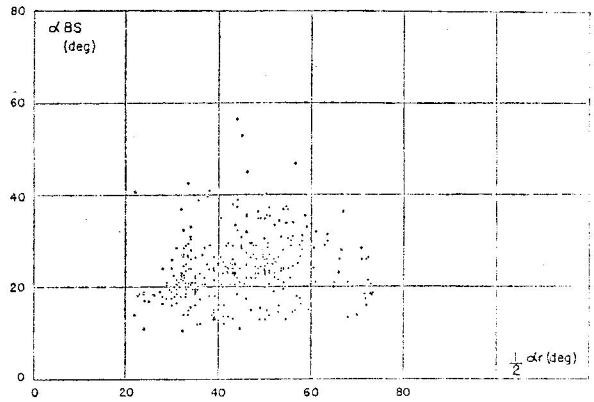


Fig 34. Relation between α_{BS}(deg.) and $\frac{1}{2}\alpha_r$(deg.) of models analysed in the computer study

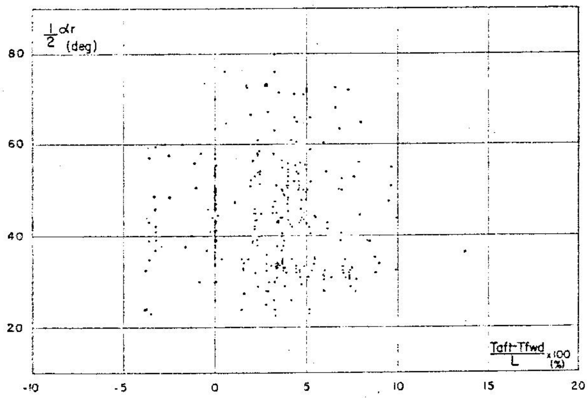


Fig 35. Relation between $\frac{1}{2}\alpha_r$(deg.) and trim of models analysed in the computer study

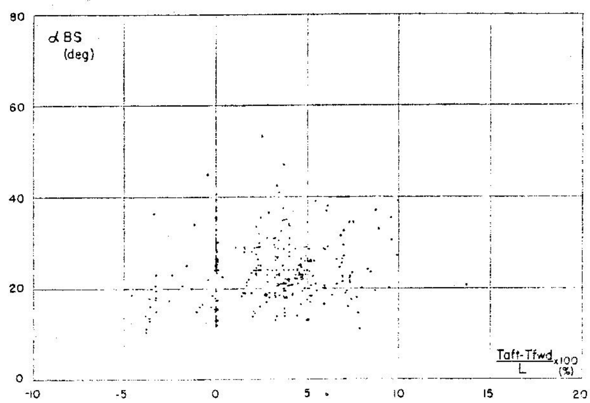


Fig 36. Relation between α_{BS}(deg.) and trim of models analysed in the computer study

pairs of form parameters are reasonably independent of each other, that is, a substantially rectangular distribution of data points is available for quite a wide range of each parameter. The extreme values of each form parameter, together with the ranges within which reasonable independence of parameters is applicable are given in table 1.

The main departure from a rectangular distribution is in the plot of C_p against $\frac{1}{2}\alpha_e^\circ$, in which there is an absence of data for high values of C_p with small values of $\frac{1}{2}\alpha_e^\circ$.

By referring to fig 1–36, it can be seen that outside the rectangular distribution of data points, there exists a considerable number of vessels, often having form parameters occurring in small groups. The importance of these forms in making performance estimates for new designs is referred to in the concluding remarks.

THE REGRESSION EQUATION FOR RESISTANCE CRITERION $C_{R_{16}}$

Having established that the parameters used to define each vessel were reasonably independent over a wide range of parameter values, it is now required to formulate equation (1) in such a manner that it conforms as closely as possible to the known or likely behaviour of each design parameter, and provides a satisfactory approximation to the data. Fortunately, some guidance was given by the earlier NPL analysis in formulating the regression equation for larger fishing vessels. The equation derived in that analysis was of polynomial type, that is, it consisted of a sum of individual terms, 30 in number, each of which was a constant multiple of a power of a parameter or of a product of such powers. In this equation, the parameters were cross-linked in pairs, though by no means all possible pairs were cross-linked. Where a pair of parameters, say X_1 and X_2, were cross-linked, the following 9 terms appeared in the equation:

$$\begin{matrix} 1 & X_1 & X_1^2 & \\ X_2 & X_1 X_2 & X_1^2 X_2 & \text{Array type (i)} \\ X_2^2 & X_1 X_2^2 & X_1^2 X_2^2 & \end{matrix}$$

This array contains all the possible cross-products of positive (or zero) powers of X_1 and X_2 when the power of each parameter is limited to a maximum of 2. It may be described as a square array of degree 2. Physically, the inclusion of this array in the equation allows for each of the two parameters to have an optimum value within its practical range, an optimum, if it exists, which is allowed to vary both in position and in sharpness with the value of the other parameter. Only six parameters (L/B, B/T, C_m, C_p, $l.c.b.$ and $\frac{1}{2}\alpha_e^\circ$) were included in this earlier analysis and the pairs of parameters which were cross-linked in the equation, in the way described (with one minor variation which is not relevant to the present discussion), were:

$$(C_p, L/B), \quad (C_p, B/T),$$
$$(C_p, l.c.b.), \quad (C_p, \tfrac{1}{2}\alpha_e^\circ),$$
and $$(L/B, \tfrac{1}{2}\alpha_e^\circ).$$

The only other term in the equation was a simple linear term in C_m.

The parameters $\frac{1}{2}\alpha_r^\circ$ and α_{BS}° were introduced in the subsequent NPL analysis of passenger-cargo vessels 1964 in order to take into account the shape of the after-body. Each of them was cross-linked with C_p in the regression equation for these vessels—the total number of terms then being 44.

These 44 terms, together with five additional ones, formed the starting point for building up the regression equation for the present analysis. These extra five terms were:—a simple linear term for trim, the two terms $(B_1 n)$ and $(B_1 n)^2$—see Section on Blockage Effects, a term to take into account the first-order effect on resistance of a wooden keel, and a term to take into account the first-order effect on resistance of omitting turbulence stimulators. In these early stages also, an attempt was made to take into account the variation of this latter effect with changes in C_p, by including an appropriate term in the regression equation, but the results obtained were unsatisfactory and the attempt was abandoned. With regard to trim, it may be repeated in passing that the effect of varying the trim of any particular hull form will be largely taken into account by the consequent modification of the other parameters, such as *l.c.b.*, and that the pure effect of trim, i.e. with all other parameters fixed, is likely to be small. This was confirmed by the analysis.

It was, of course, clear that this initial equation of 49 terms would need considerable expansion before a satisfactory fit to the data could be achieved: the ranges of the parameters in the FAO data are substantially wider than in the NPL data and so terms that were negligible in the latter case could become effective in the former case. Nevertheless, it was helpful to have a nucleus of terms, known to be important, round which to build. Fortunately, the much larger number of models available in the FAO data (over 300), made it possible to contemplate such a major expansion. On the other hand, the total number of possible combinations of 8 or 9 parameters up to, say, degree 4, is also very large, and so it was still necessary to be very selective in deciding which terms to add to the initial equation.

The general procedure adopted was to add to the equation, a few at a time, new terms which were considered likely to be effective, to fit the extended equation to the data, and then to assess the effectiveness of the new terms by considering the improvement in the closeness of fit. The purpose of the fitting procedure is to determine the best values of the constant multiples occurring in the equation, the best values, that is, in the sense that they minimize the sum of squares of the differences between the data values of C_R and the values calculated from the equation. (These differences are the "residuals".) Thus the fitting procedure was carried out using the usual least-squares criterion. With equations of the size we are considering, a great deal of computation is involved and this was carried out on the ACE computer at NPL. The improvement in the closeness of fit was assessed by consideration of the residuals: if the addition of the new terms to the equation resulted in a satisfactory reduction in the sum of squares of the residuals, the new terms were accepted as part of the final equation, otherwise they were rejected.

In considering ways of expanding the regression equation, there were three main directions we could contemplate:

- Pairs of parameters could be cross-linked which were not already cross-linked
- Products containing more than two parameters could be introduced
- Pairs of parameters which were already cross-linked could be taken to a higher power, e.g., the square array of type (i) could be increased from degree 2 to degree 3

Each of these three ways had to be considered. In the first category, the parameter which stood out as requiring further attention was the maximum area coefficient C_m. In the original NPL analysis, this parameter was found to be relatively unimportant, but in the present data C_m varies between 0.44 and 0.88, a very wide range which represents very substantial differences in the character of the hull forms. Consequently C_m was introduced into the equation cross-linked with both L/B and B/T, and this was found to be beneficial. In the second category, four major parameters, L/B, B/T, C_p and $\frac{1}{2}\alpha_e^\circ$, were considered and the four triple products obtained by multiplying these parameters together three at a time were introduced into the equation. The result was unsatisfactory and so it was decided to consider only terms containing not more than two parameters.

At about this stage in the analysis, it was decided that, instead of the basic square array of type (i), which limits the power of each parameter separately, it would be more logical to use an array which placed a limit on the combined powers of the two parameters concerned. Consequently, the regression equation was modified so that, for each cross-linked pair of parameters, e.g., X_1 and X_2, the equation contained the following 10 terms:—

1	X_1	X_1^2	X_1^3	
X_2	$X_1 X_2$	$X_1^2 X_2$		Array type (ii)
X_2^2	$X_1 X_2^2$			
X_2^3.				

This array contains all the possible cross-products of positive (or zero) powers of X_1 and X_2 when the sum of the powers of the two parameters is limited to a maximum of 3. It may therefore be described as a triangular array of degree 3. In effect, it removes the term $X_1^2 X_2^2$ from the square array of degree 2 and adds the two terms X_1^3 and X_2^3. Physically, as before, this array allows for each of the two parameters to have an optimum value which varies both in position and in sharpness with the value of the other parameter, and at the same time allows for some departure from the purely quadratic form of the previous array.

At this point the equation was still some way from giving a satisfactory fit to the data, and so we undertook an extensive investigation into possible additional terms in categories above. As a result of this, the following new pairs of parameters, cross-linked to degree 3 were added to the equation:

$$(B/T, \tfrac{1}{2}\alpha_e^\circ),\quad (l.c.b., \tfrac{1}{2}\alpha_e^\circ),$$
$$(L/B, \alpha_{BS}^\circ),\quad (B/T, \alpha_{BS}^\circ),\quad (l.c.b., \alpha_{BS}^\circ).$$

Also, the fourth degree terms of the following pairs of parameters, already cross-linked to degree 3, were added to the equation so that these pairs became cross-linked to degree 4:

$$(B/T, C_m),\quad (L/B, \tfrac{1}{2}\alpha_e^\circ),\quad (C_p, L/B).$$

The fourth degree terms of the following pairs were similary tested, but rejected as unnecessary:

$$(L/B, C_m),\quad (C_p, B/T),\quad (C_p, l.c.b.),$$
$$(C_p, \tfrac{1}{2}\alpha_e^\circ),\quad (C_p, \tfrac{1}{2}\alpha_r^\circ),\quad (C_p, \alpha_{BS}^\circ).$$

These pairs were therefore kept in the equation cross-linked to degree 3. Finally, three terms in the block coefficient $C_B=(C_p\times C_m)$ were tested but rejected, namely: C_B, C_B^2 and C_B^3.

At this stage, the standard error (root mean square) of the residuals for the data at $V/\sqrt{L}=1.0$ was down to 0.74 corresponding to a basic scatter about the fitted expression of approximately $\pm 3\frac{1}{2}$ per cent of the average C_R value. Consequently, the equation was accepted as satisfactory. The final form of the regression equation contains 86 terms and is given in full in the Appendix.

All the above work of building up the regression equation was carried out on the data for $V/\sqrt{L}=1.0$. Once the final form of equation was settled, it was fitted to the data for each of the other values of $V/\sqrt{L}$ for which there was a sufficient number of models to yield a satisfactory result, namely $V/\sqrt{L}=0.85$ and then at intervals of 0.05 up to 1.20. There was an insufficient number of models at $V/\sqrt{L}=0.7$, 0.75 and 0.80.

During the work of building up the equation for $V/\sqrt{L}=1.0$, a check was kept on models which persistently gave high residuals. These could be genuine departures from the regression equation, since a number of larger residuals must be expected simply because of random variations, but it was also possible that there was some extraneous reason why the equation could not be expected to fit a particular model, possibly because it contained some unusual feature which was not taken into account, and in this case its inclusion in the fitting process might unnecessarily distort the fit. Consequently, all the models with persistently high residuals were referred back to FAO for investigation into the detailed records, and where there was a satisfactory reason to explain a poor result, the model was rejected from the analysis. The rejected models included, for example, cases where the bar keel had been carried up to the waterline forward, and had not been tapered off, cases where the running trim differed abnormally from the static trim, so that the parameters used were not applicable to running conditions, and cases which squatted an abnormal amount. There were also six models with bulbous bows, which, as in the previous NPL work, were not fitted satisfactorily by the equation. Then, of course, there were the definite human errors which are bound to occur in a collection of data of this magnitude. These were picked out by their gross inconsistency with other data, either with the same model at other speeds or with other models having almost identical parameters. In all 32 models were rejected for one such reason or another. The total number of models remaining after rejecting these 32 is given for each value of $V/\sqrt{L}$ in table 2, together with the root-mean-square of residuals.

TABLE 2

$V/\sqrt{L}$	0.85	0.90	0.95	1.00	1.05	1.10	1.15	1.20
Number of models	184	222	245	249	245	240	229	196
Standard error (root mean square) of residuals of $C_{R_{16}}$	0.55	0.64	0.68	0.74	0.75	0.83	0.94	1.05

ESTIMATION OF PERFORMANCE FOR PARTICULAR HULL FORMS

Having determined the regression equations for $C_{R_{16}}$ at a series of values of speed-length ratio from $V/\sqrt{L}=0.85$ to $V/\sqrt{L}=1.20$, an auxiliary computer program was prepared to evaluate resistance performance for any required combination of hull form parameters. This program was written for the KDF 9 Computer in Mathematics Division, NPL, and requires as input data the regression coefficients $a_0\ a_1\ a_2 \ldots a_{85}$ together with the numerical specification of parameters $[X_1\ X_2\ X_3 \ldots X_9]$ and the individual terms such as X_1X_2, X_4^4, $X_2^3X_3$, etc., of which the regression equation is composed. The program can be used if required to evaluate other regression equations of this type up to the ninth power in 13 variables, including the constant term a_0.

In addition to evaluating the values of $C_{R_{16}}$ for each speed-length ratio, the program also determines the values of C_R at any required ship length (L) together with the corresponding values of EHP using the ITTC formulation, i.e.,

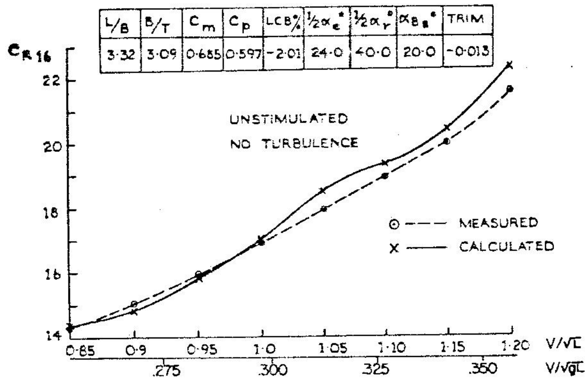
$$C_{R(L)}=C_{R(16)}-0.212847\left(\frac{S\cdot L}{\Delta}\right)\left[\left(\log 88\cdot\frac{V}{\sqrt{L}}\cdot 10^3\right)^{-2}-\left(\log 1.2834\frac{V}{\sqrt{L}}.L^{\frac{3}{2}}\cdot 10^3\right)^{-2}\right] \quad (7)$$

where S=wetted hull surface area (ft^2)
L=Length on waterline (ft)
Δ=ship displacement (moulded) in tons (35 ft^3/ton)
V=ship speed (knots)

and

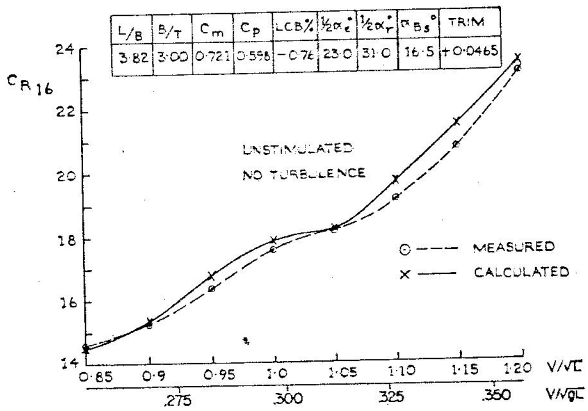
$$\text{EHP (using ITTC formulation)}=\frac{C_{R(L)}\cdot\Delta\cdot V^3}{325.7L}. \quad (8)$$

To illustrate the use of the computer program and the types of $C_{R_{16}}$ curves which are obtained for particular combinations of parameters, estimates of performance have been made for seven forms covering a fair range of each parameter, and are plotted against $V/\sqrt{L}$ in fig 37 to 43. In the case of Model Nos 40 and 48 (fig 37 and 38),



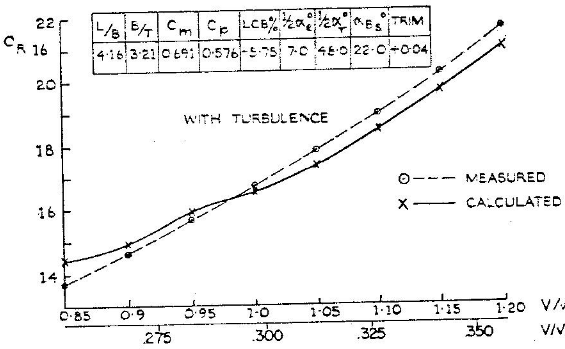
L/B	B/T	C_m	C_p	LCB%	$\frac{1}{2}\alpha_e°$	$\frac{1}{2}\alpha_r°$	$\alpha_{Bs}°$	TRIM
3·32	3·09	0·685	0·597	−2·01	24·0	40·0	20·0	−0·013

Fig 37. Model No. 40



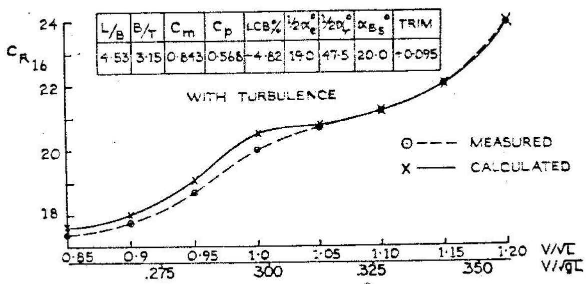
L/B	B/T	C_m	C_p	LCB%	$\frac{1}{2}\alpha_e°$	$\frac{1}{2}\alpha_r°$	$\alpha_{Bs}°$	TRIM
3·82	3·00	0·721	0·598	−0·76	23·0	31·0	16·5	+0·0465

Fig 38. Model No. 48



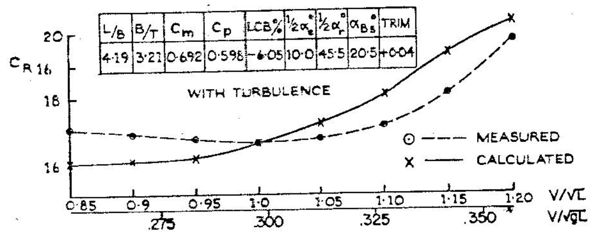
L/B	B/T	C_m	C_p	LCB%	$\frac{1}{2}\alpha_e°$	$\frac{1}{2}\alpha_r°$	$\alpha_{Bs}°$	TRIM
4·16	3·21	0·691	0·576	−5·75	7·0	48·0	22·0	+0·04

Fig 39. Model No. 199



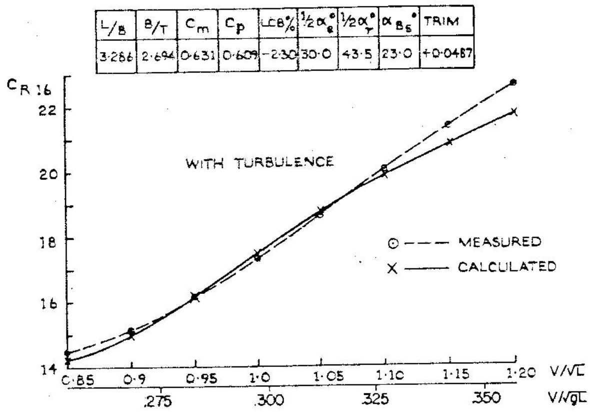
L/B	B/T	C_m	C_p	LCB%	$\frac{1}{2}\alpha_e°$	$\frac{1}{2}\alpha_r°$	$\alpha_{Bs}°$	TRIM
4·53	3·15	0·843	0·568	−4·82	19·0	47·5	20·0	+0·095

Fig 40. Model No. 203



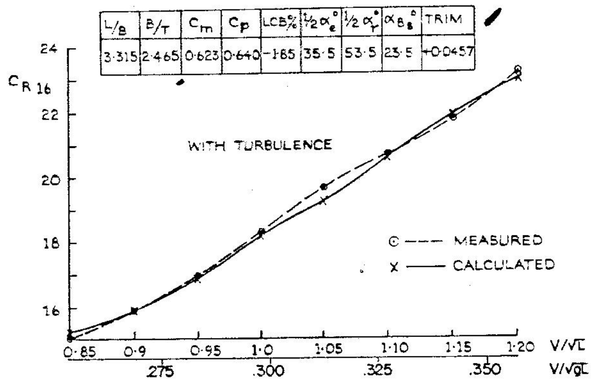
L/B	B/T	C_m	C_p	LCB%	$\frac{1}{2}\alpha_e°$	$\frac{1}{2}\alpha_r°$	$\alpha_{Bs}°$	TRIM
4·19	3·21	0·692	0·598	−6·05	10·0	45·5	20·5	+0·04

Fig 41. Model No. 193



L/B	B/T	C_m	C_p	LCB%	$\frac{1}{2}\alpha_e°$	$\frac{1}{2}\alpha_r°$	$\alpha_{Bs}°$	TRIM
3·286	2·694	0·631	0·609	−2·30	30·0	43·5	23·0	+0·0487

Fig 42. Model No. 2021



L/B	B/T	C_m	C_p	LCB%	$\frac{1}{2}\alpha_e°$	$\frac{1}{2}\alpha_r°$	$\alpha_{Bs}°$	TRIM
3·315	2·465	0·623	0·640	−1·65	35·5	53·5	23·5	+0·0457

Fig 43. Model No. 2004

estimates have been made for the case where no turbulence stimulators were fitted, corresponding to the actual test conditions for these models. Although the measured results are less stable than those obtained with turbulence stimulators fitted to the model, it can be seen that the estimates obtained using the regression equation are in good agreement. It should also be noted that each estimate of $C_{R_{16}}$ is independently derived and that even so the general character of the resistance curve in terms of speed-length ratio is reasonably well simulated. Models 199 and 203 (figs 39 and 40), both having turbulence stimulators fitted, again show good general agreement between the measured and calculated resistance-speed curves, although the "hump" at $V/\sqrt{L}=1.00$ has been somewhat suppressed in the case of Model 203. Model 193 (fig 41) shows the largest differences between the measured and calculated results, although the two curves agree at $V/\sqrt{L}=1.00$. There is some evidence to suggest that the measured results below $V/\sqrt{L}=1.00$, which show a rising characteristic as speed reduces, may be suspect due to over-stimulation, and the estimated curve of $C_{R_{16}}$ is certainly more in keeping with practical experience. Models 2021 and 2004 (fig 42 and 43) are derived from a separate Tank and are again reasonably well fitted by the regression equation.

SOME EFFECTS ON RESISTANCE CRITERION OF INDIVIDUAL PARAMETERS

The effects on resistance criterion $C_{R_{16}}$ due to changes in individual hull form parameters are rather complex and vary both with speed-length ratio and the values of the other parameters. In order to give some guidance to designers of the smaller fishing vessels, fig 44 to 51 have been prepared to show several effects of individual parameters on resistance criterion for central values of the remainder at $V/\sqrt{L}=1.10$. It was therefore considered a basic form having the following hull form parameters and discrete changes were made in each parameter.

The hull form parameters of the basic form are:

$[L/B=3.75,\ B/T=2.75,\ C_m=0.65,\ C_p=0.61,\ l.c.b.\%=-2.0,\ \frac{1}{2}\alpha_e^\circ=25.0,\ \frac{1}{2}\alpha_r^\circ=45.0,\ \alpha_{BS}^\circ=25.0,\ \text{trim}=+0.05]$.

(a) L/B ratio The effect of length-beam ratio has been studied between 3.1 to 4.2 at various values of prismatic coefficient between $C_p=0.55$ to $C_p=0.65$, for fixed values of the remaining parameters of the basic form.

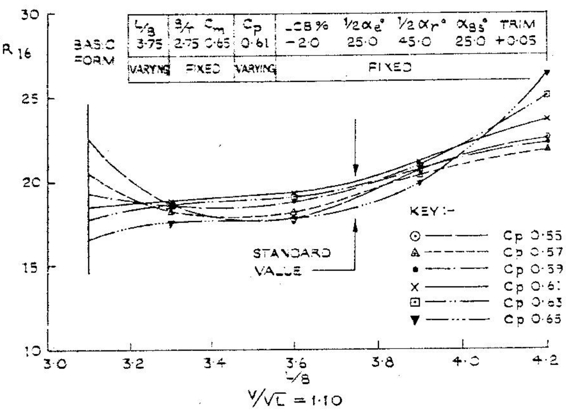


Fig 44. The effects of variations in L/B ratio from the standard value of 3.75 for various prismatic coefficients

Fig 44 shows that reduction in L/B ratio down to a value of approximately 3.40 is generally beneficial for all prismatic coefficients. Below values of $C_p=0.59$ however there appears to be some penalty in resistance criterion for L/B ratios less than 3.40.

(b) B/T ratio The effect of beam-draft ratio for fixed values of the remaining parameters has been studied for B/T ratios between 2.0 and 3.2 and for C_p values between 0.55 to 0.65. Fig 45 shows that increase in B/T ratio always results in a penalty in resistance criterion for all values of prismatic coefficient. With the exception of $C_p=0.65$, which is on the edge of the rectangular range of data points included in the analysis, the penalty change in resistance criterion due to changes in B/T ratio is generally the same for all prismatic coefficients.

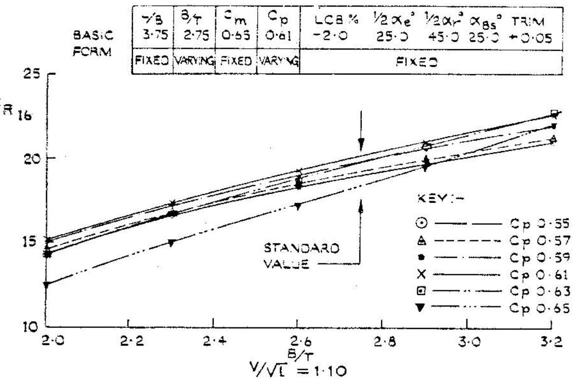


Fig 45. The effects of variations in B/T ratio from the standard value of 2.75 for various prismatic coefficients

(c) C_m The effects of variations in maximum area coefficient C_m have been calculated over the range $C_m=0.50$ to $C_m=0.80$. This large range covers a wide variety of hull forms from the yacht-shaped hulls with fine midship sections up to the fuller-sectioned and generally larger fishing vessels around 100 ft in length. As can be seen from fig 46

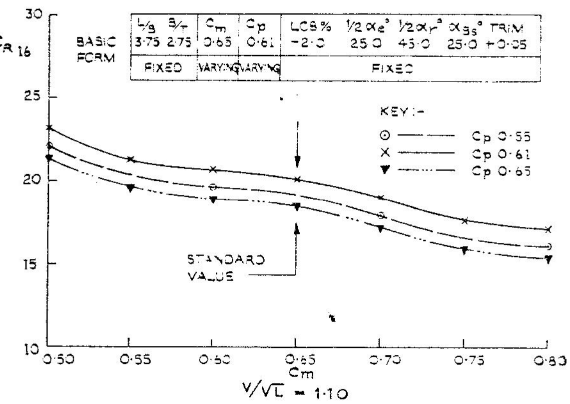


Fig 46. The effects of variations in maximum section area coefficient from the standard value of $C_m=0.65$

there is a penalty in resistance criterion as C_m is reduced and this penalty is the same for all values of prismatic coefficient within the range $C_p=0.55$ to $C_p=0.65$. The slope of the $C_{R_{16}}$-C_m curve, however, is rather less for values of C_m in the region of $C_m=0.75$ to $C_m=0.80$, suggesting that for these fuller sections the benefits of further increase in maximum section area are not so great.

(d) C_p The effects on resistance criterion of changes in prismatic coefficient from the standard value of $C_p=0.61$ have been calculated for fixed values of the remaining parameters of the basic form. It can be seen from fig 47 that a prismatic coefficient of 0.61 is about the worst value for the basic form and that both higher and lower values are advantageous.

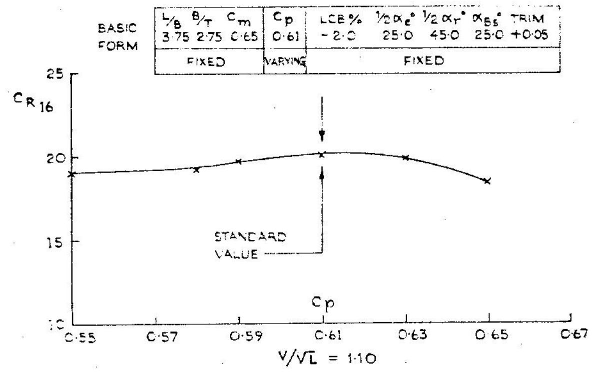


Fig 47. The effects of variations in prismatic coefficient from the standard value of $C_p=0.61$

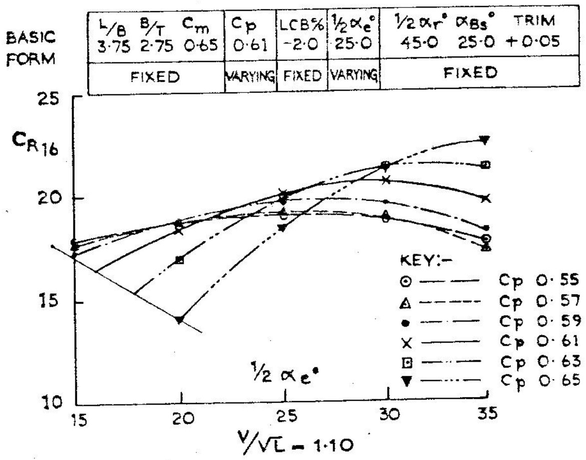


Fig 49. The effects of variations in half-angle of entrance of the floating waterline from the standard value of $\frac{1}{2}\alpha_e^\circ = 25^\circ$ or various prismatic coefficients

(e) *l.c.b.*% The effects on resistance criterion of changes in position of the longitudinal centre of buoyancy can be seen in fig 48 for values of prismatic coefficient between $C_p=0.55$ to $C_p=0.65$. It can be seen that there is generally a benefit in locating the position of the *l.c.b.* up to 6.0 per cent aft of amidships, although for C_p values in excess of 0.63 the optimum position appears to be at or near amidships.

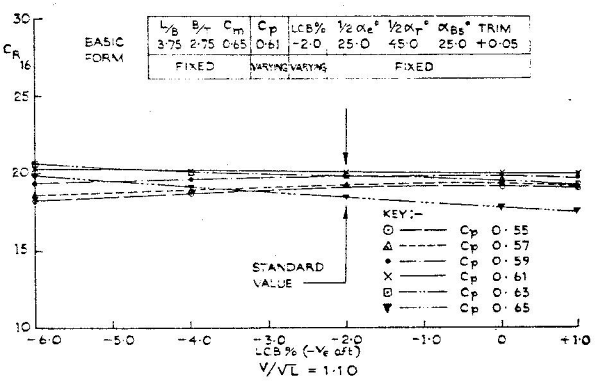


Fig 48. The effects of variations in position of longitudinal centre of buoyancy from the standard value of −2.0 per cent for various prismatic coefficients

(f) $\frac{1}{2}\alpha_e^\circ$ The effects on resistance criterion of changes in the half-angle of entrance of the load waterline are very marked as can be seen in fig 49. Generally speaking, the advantages of adopting a low value of $\frac{1}{2}\alpha_e^\circ$ are apparent at all values of prismatic coefficient, although for C_p values of 0.55 and 0.57 it appears that almost equally good performance can be obtained for values of $\frac{1}{2}\alpha_e^\circ$ between 30° and 35° as for values of $\frac{1}{2}\alpha_e^\circ$ between 15° and 20°. Both of these ranges of $\frac{1}{2}\alpha_e^\circ$ are better than the standard value of 25° for the basic form.

(g) $\frac{1}{2}\alpha_r^\circ$ The effects on resistance criterion of variations in half-angle of run from the standard value of $\frac{1}{2}\alpha_r^\circ=45^\circ$ for various prismatic coefficients, are of secondary importance in relation to the remaining form parameters (see fig 50). For the ranges of hull form parameters with which we are concerned in our analysis, therefore, it is generally permissible in design work to allow the run angles of the form to increase if required to make the buttock angles less steep.

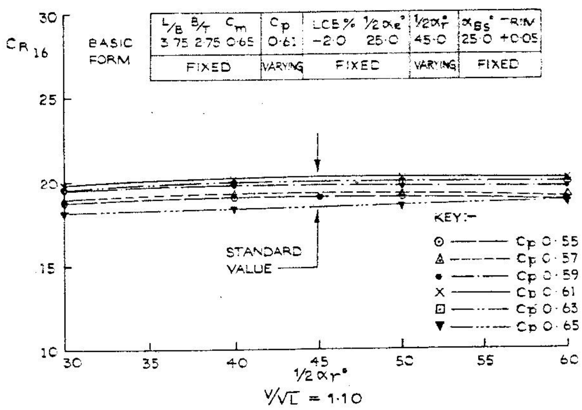


Fig 50. The effects of variations in half-angle of run from the standard value of $\frac{1}{2}\alpha_r^\circ = 45^\circ$ for various prismatic coefficients

(h) α_{BS} The effects on resistance criterion of variations in buttock slope are generally significant for all values of prismatic coefficient and show a benefit in $C_{R_{16}}$ as buttock slope is reduced down to 15° (the lower limit of the data). These effects can be seen in fig 51.

(i) trim The effect on resistance criterion of changes in trim from the design value of +0.05 are not significant.

It is emphasized that the independent changes in L/B, B/T, C_m and C_p discussed above will all affect displacement. In practice, if changes are investigated which keep length and displacement constant, a change in

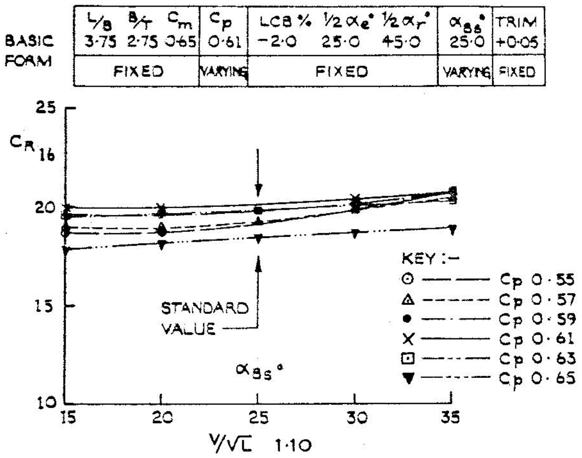


Fig 51. The effects of variations in buttock slope from the standard value of $\alpha^{\circ}_{BS} = 25^{\circ}$ for various prismatic coefficients

any one of these four parameters will necessitate appropriate changes in one or more of the other three, and the several resulting effects on C_R will counteract or reinforce one another, thus modifying the above conclusions in particular cases.

CONCLUSION

Similar trends showing the influence of hull form parameters at all speed-length ratios between $V/\sqrt{L}=0.85$ to $V/\sqrt{L}=1.20$ can be derived from the computer program. As we have demonstrated however, the effects on resistance criterion are quite complex. Estimates for individual vessels and suggestions for improvement in performance should therefore be made using the computer program as a design tool.

This analysis covers a wide range of hull form parameters and fishing vessel types. As more data sheets for new vessels are included in the FAO store of information however, it should be possible to extend the coverage of the present analysis even further. The process is therefore seen as one of continuous appraisal, modification and improvement in the design of fishing vessels.

The expressions in the regression equations derived from this analysis can be explored in order to seek combinations of hull form parameters which will give improved performance. This exploration can be carried out either by some form of systematic evaluation of the expressions or by using one of the methods of mathematical optimization which are currently available. Four different sets of hull form parameters have been derived by these methods and forms having these parameters have been designed by FAO and Chalmers Technical University, Göteborg, and models have been tested at NPL and Chalmers. This work is presented on page 139.

As already noted, there are a considerable number of vessels included in the present analysis having hull form parameters outside the ranges given in table 1. For new vessels which have hull form parameters in these regions of existing data, satisfactory estimates of performance can usually be made using the regression equation given in the Appendix.

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Kungl. Tekniska Högskolan, Stockholm, Sweden
Swedish State Shipbuilding Experimental Tank, Göteborg, Sweden
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National Physical Laboratory, Teddington, Middlesex, UK.
Davidson Tank, Stevens Institute of Technology, Hoboken, New Jersey, USA
The Naval Tank, Department of Naval Architecture and Marine Engineering, University of Michigan, Ann Arbor, USA
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NOMENCLATURE

L	length on the floating waterline. Obvious long stems, etc., are faired out and the appropriate length determined. In some cases L is determined from drawings, as length between perpendiculars is sometimes the only length given in reports
B	beam, maximum, usually at the midlength of dimension L, measured at the floating waterline WL
T_{aft}	draft at the aft end of L to an elongation of the centre bottom line of a steel hull (excluding a possible bar keel) or to the elongated rabbet line of a straight keel of a wood hull
T	draft moulded at $\frac{1}{2}L$
T_{fwd}	draft at the fore end of L to an elongation of the centre bottom line of a steel hull (excluding a possible bar keel) or to the elongated rabbet line of a straight keel of a wood hull

A_m — maximum immersed section area (the area of the vertical transverse underwater section of the model which has the greatest section area)

∇_m — volume of displacement of the model up to the floating waterline (ft^3)

∇ — metric displacement volume of the underwater body of the vessel, including keel and appendages to waterline *WL* in $(m)^3$

Δ — displacement of ship in salt water, floating at waterline *WL*, based on 35 ft^3 of salt water per ton, corresponding to a specific gravity of 1.026 in long tons of 2,240 lb (1,016 kg)

S — wetted area of the underwater body of the model to waterline *WL*. This includes the wetted surface of all appendages in the appendage list at the top of the FAO data sheets, excluding struts and open shafts

V — ship speed in knots

v — ship speed in ft/sec

ν — coefficient of kinematic viscosity; assumed to be 1.2285×10^{-5} for model at 59°F in fresh water and 1.316×10^{-5} for ship at 59°F (15°C) in salt water

R_N — Reynolds number $= (vL/\nu)$

$v/\sqrt{gL}$ — Froude number

$V/\sqrt{L}$ — speed-length ratio

C_m — maximum area coefficient evaluated to waterline *WL* ($C_m = A_m/BT$)

C_p — prismatic coefficient based on the maximum section area and the moulded displacement including stern ($C_p = 35\Delta/L.A_m$)

$C_B = C_m \cdot C_p$ — block coefficient: the volume of the underwater body of the ship divided by the volume of a rectangular block having dimensions *L*, *B*, *T* as the ship

L/B — length-beam ratio

B/T — beam-draft ratio

$l.c.b.\%$ — position of the longitudinal centre of buoyancy in relation to the midlength of the dimension *L*, expressed as a percentage of the length *L*

+ indicates the position forward of $\frac{1}{2}L$ and

− indicates the position aft of it

$\frac{1}{2}\alpha_e^\circ$ — the angle which the waterline *WL* makes with the centreline of the model at the stem. Normally this is the average angle for the first $\frac{1}{20}$ of the length *L*, but when measuring, care is taken to disregard excessive rounding or hollowing near the stem.

$\frac{1}{2}\alpha_r$ — the maximum angle of run up to end including the designed floating waterline *WL*. This angle is measured at a section 5 per cent of the waterline length forward of the after end of *L*, except where this section cuts the deadwood, when the maximum waterline slope at the intersection with the forward end of the deadwood is taken.

α_{BS}° — the maximum buttock slope of the $\frac{1}{4}$ beam buttock measured relative to the floating waterline *WL*. This angle is evaluated *exclusive* of the slope of the stern contour in the case of vessels with transom sterns

R — ship resistance in lb

$C_{R(L)} = \frac{R \cdot L}{\Delta V^2}$ — resistance criterion proposed by Telfer

C_{R16} — resistance criterion when $L = 16$ ft (4.9 m)

R_f — three-dimensional frictional resistance component given by ITTC Formulation

ρ — relative density or specific gravity

v_m — model speed of advance in ft/sec

l — length of model in ft corresponding to ship length *L*

b — breadth of model in ft corresponding to ship breadth B or breadth of water in tank (Formula 3)

h — depth of water in Tank (ft)

g — acceleration due to gravity (ft/sec^2)

APPENDIX

Final Regression Equation

$$
\begin{aligned}
C_{R_{16}} = {} & a_0 + a_1 X_1 + a_2 X_2 + a_3 X_3 + a_4 X_4 + a_5 X_5 + a_6 X_6 + a_7 X_7 + a_8 X_8 + a_9 X_9 \\
& + a_{10} X_1^2 + a_{11} X_2^2 + a_{12} X_3^2 + a_{13} X_4^2 + a_{14} X_5^2 + a_{15} X_6^2 + a_{16} X_7^2 + a_{17} X_8^2 \\
& + a_{18} X_1^3 + a_{19} X_2^3 + a_{20} X_3^3 + a_{21} X_4^3 + a_{22} X_5^3 + a_{23} X_6^3 + a_{24} X_7^3 + a_{25} X_8^3 \\
& + a_{26} X_1^4 + a_{27} X_2^4 + a_{28} X_3^4 + a_{29} X_4^4 + a_{30} X_6^4 \\
& + a_{31} X_2 X_3 + a_{32} X_2^2 X_3 + a_{33} X_2 X_3^2 + a_{34} X_2^3 X_3 + a_{35} X_2^2 X_3^2 + a_{36} X_2 X_3^3 \\
& + a_{37} X_1 X_6 + a_{38} X_1^2 X_6 + a_{39} X_1 X_6^2 + a_{40} X_1^3 X_6 + a_{41} X_1^2 X_6^2 + a_{42} X_1 X_6^3 \\
& + a_{43} X_1 X_4 + a_{44} X_1^2 X_4 + a_{45} X_1 X_4^2 + a_{46} X_1^3 X_4 + a_{47} X_1^2 X_4^2 + a_{48} X_1 X_4^3 \\
& + a_{49} X_2 X_4 + a_{50} X_2^2 X_4 + a_{51} X_2 X_4^2 \\
& + a_{52} X_4 X_5 + a_{53} X_4^2 X_5 + a_{54} X_4 X_5^2 \\
& + a_{55} X_4 X_6 + a_{56} X_4^2 X_6 + a_{57} X_4 X_6^2 \\
& + a_{58} X_4 X_7 + a_{59} X_4^2 X_7 + a_{60} X_4 X_7^2 \\
& + a_{61} X_4 X_8 + a_{62} X_4^2 X_8 + a_{63} X_4 X_8^2 \\
& + a_{64} X_1 X_3 + a_{65} X_1^2 X_3 + a_{66} X_1 X_3^2 \\
& + a_{67} X_2 X_6 + a_{68} X_2^2 X_6 + a_{69} X_2 X_6^2 \\
& + a_{70} X_5 X_6 + a_{71} X_5^2 X_6 + a_{72} X_5 X_6^2 \\
& + a_{73} X_1 X_8 + a_{74} X_1^2 X_8 + a_{75} X_1 X_8^2 \\
& + a_{76} X_2 X_8 + a_{77} X_2^2 X_8 + a_{78} X_2 X_8^2 \\
& + a_{79} X_5 X_8 + a_{80} X_5^2 X_8 + a_{81} X_5 X_8^2 \\
& + a_{82}(B_1 n) + a_{83}(B_1 n)^2 + a_{84}\delta_1 + a_{85}\delta_2
\end{aligned}
$$

where

$X_1 = L/B$

$X_2 = B/T$

$X_3 = C_m$

$X_4 = C_p$

$X_5 = l.c.b.$

$X_6 = \frac{1}{2}\alpha_e^\circ$

$X_7 = \frac{1}{2}\alpha_r^\circ$

$X_8 = \alpha_{BS}^\circ$

$X_9 = \text{trim}$

$$\delta_1 = \begin{cases} 0, & \text{if there is no wooden keel} \\ 1, & \text{if there is a wooden keel} \end{cases}$$

$$\delta_2 = \begin{cases} 0, & \text{if turbulence stimulators are fitted} \\ 1, & \text{if turbulence stimulators are not fitted} \end{cases}$$

$a_0, a_1, a_2, \ldots, a_{85}$ are constants determined by the least-squares fitting, a different set for each value of $V/\sqrt{L}$.

a ⇒ Section area of a bar or wooden keel in ft²
Amax ⇒ Max immersed section area ft²
B ⇒ ft
R = lb
S = ft²
V ⇒ knot