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Hull Resistance

COMPUTER-AIDED STUDIES OF FISHING BOAT HULL RESISTANCE

by

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PREPARATION OF THIS DOCUMENT

FAO has since 1950 devoted considerable attention to questions related to the resistance and powering of fishing vessels because it was felt that findings from one type of fishing vessel could easily be applied to a completely different type.

A method to evaluate the results of model tests statistically, developed at the National Physical Laboratory (NPL) of the United Kingdom, was applied to the large number of test results for small fishing vessels which had been collected at FAO. Reviews of the FAO work on hull resistance have been published in connexion with FAO International Fishing Boat Congresses 1953, 1959 and 1965 and this document is the result of the most recent work on the material.

The work of one of the authors (Hayes) was carried out as part of the research programme of the National Physical Laboratory. The computational work, including computer programming, entailed in the derivation of the regression equations and the production of the tables was performed by Mr. G.T. Anthony (NPL).

Mr. T. Tsuchiya, Fishing Boat Laboratory, Tokyo, has assisted in the examination and processing of the Japanese data included in the study.

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1. SUMMARY

The first results of the joint NPL/FAO computer study of FAO resistance data for fishing craft were described at the Third FAO Fishing Boat Congress at Göteborg in 1965. These enabled hull resistance to be estimated from a number of parameters of hull shape and dimensions, within certain ranges of these parameters. This work utilized only part of the FAO data available, since problems had been encountered with the remainder. These difficulties appear now to have been overcome and the new equations obtained are applicable to wider parameter regions, thus giving increased practical utility. Moreover, the specification of these regions is the result of a more detailed investigation than previously and should therefore be more reliable.

It is therefore considered appropriate now to publish full numerical details of the equations and regions so that other workers can make use of the results. They should, however, be used with due discretion: certainly, estimates cannot be relied upon when one or more parameters lie outside the defined region of validity, though this applies more strongly to some parameters than to others, as can be seen from the tables. Moreover, conclusions on optimum forms, since their parameters tend to lie on the edges of the defined region, should preferably be checked by a model test. It is hoped that such other workers will inform FAO of their experiences in using the results presented so that further information on their reliability can be rapidly accumulated, particularly in relation to the specification of the parameter regions. Results of model tests will be especially valuable. If sufficient additional data can be thus accumulated, there will be the possibility in due course of adding them to the analysis, so making the analysis even more comprehensive and reliable.

2. BACKGROUND

One of the main deterrents to systematic progress in fishing vessel design for smaller vessels (below 100 ft [30 m] in length), and the determination of their resistance and propulsive qualities, has always been the relatively high cost of a model text in relation to the capital cost of the vessel. Another related consideration is that vessels of this category may vary tremendously in main proportions and hull form characteristics, thereby prohibitively increasing the cost of conducting any systematic experimental programme.

FAO has, for a long time, collected results of model tests on fishing vessels in many countries (see, for example, Traung 1955, 1959) and has also sponsored a number of such tests. The question thus arose of whether some comprehensive evaluation of these data could be made, in order to make it available in a form which would be of general value to all Member Governments.

Doust (1963) and Hayes (1964) had made such an evaluation, based on statistical regression analysis of a corresponding, though smaller, set of NPL data for trawlers. Consequently, NPL was requested by FAO to investigate the practicability of performing a similar analysis with the FAO data. This subsequently became a joint FAO/NPL study, and the first results were presented at the Third FAO Fishing Boat Congress in Göteborg, 1965 (Doust, Hayes, Tsuchiya, 1967). These show how the hull resistance (in terms of the coefficient C_{RL} based on a 16-ft model) can be expressed by a regression equation containing 86 terms which are powers, and products of powers, of 12 parameters. These 12 parameters are those listed in Appendix I, with the omission of a/A_{max} (keel cross-sectional area/maximum transverse section area). Nine of the parameters are various measures of hull shape and dimensions, one is concerned with the presence or otherwise of a keel, one is a blockage correction for the finite size of the experimental tank, and the last is a correction for cases when turbulence stimulators were not fitted to the model.

There is one such regression equation for each of a number of values of speed-length ratio ($V/\sqrt{L}$), and the equations are applicable over certain parameter ranges depending on the data used. The accuracy of representing the resistance (C_{R16}) by means of the equations was found to be satisfactory, as the residuals (CR_{16} calculated - CR_{16} measured) had a standard error (root mean square) of only 4 per cent of the average CR_{16} value, for speed-length ratios in the region 1.0 to 1.1.

A particularly important use of the equations is to identify parameter combinations which will give low-resistance characteristics. Also, if parameter values are restricted through local geographical or climatic conditions, or for other technical considerations, it is possible to assess the penalties in performance incurred because of the restriction.

In order to test the validity of the analysis, it was decided to design and test four hulls of different sizes. It was considered desirable to make these designs as good as possible hydrodynamically in order to indicate ways in which small and medium-sized fishing boats could be improved. To derive the parameters of the hulls a systematic tabulation of the equation for speed-length ratic 1.1 was used to carry out a limited optimization. Full details and results for these models were also presented at the Göteborg Congress (Traung, Doust, Hayes, 1967).

Data from 276 model tests were used in the above regression analysis. All these model tests had been carried out in European tanks, mainly the Swedish State Shipbuilding Experimental Tank. In addition, data were available on 337 model tests from the Fishing Boat Laboratory in Tokyo. It had been intended to combine both sets of data into a single regression analysis with a view to deriving equations valid over wider parameter ranges, thus increasing their practical utility. However, the form of equation developed with the European data did not fit the Japanese data so closely, the standard error of the residuals for the data at $V/\sqrt{L} = 1.0$ being about 6 per cent of the average CR_{16} value. Therefore the fitting to the combined data was postponed until the reasons for this difficulty had been clarified.

Further consideration has now been given to the question, and this report describes the results achieved. It gives the full details necessary for others to apply the results in practice.

3. JAPANESE DATA

Certain of the Japanese models had given unusually high residuals, and it was decided to re-examine these forms and their test results in detail in an attempt to discover any unusual design features which might have affected the results, or any errors in the data. It was concluded that:

1. The keel factor being used (which simply allowed a constant increment in CR_{16} when a keel of any size was present) was inadequate in view of the large range in keel size encountered (a up to 7½ per cent of A_{max}), and that it should be replaced by a/A_{max} and $(a/A_{max})^2$.

2. Thirteen models with various special individual features which would have affected the resistance should be discarded, as also should four other models because of errors.

The implementation of these conclusions reduced the standard error of residuals by about 10 per cent of its previous value, most of the improvement being due to the discarded data, though the new keel term did have some effect.

In view of these findings, it was decided to carry out a similar detailed re-examination of all the other data. While this was taking place, a further possibility was investigated, namely, that because of the much wider ranges of some of the parameters in the Japanese data, additional terms might be necessary in the regression equation. In this investigation, no new parameter combinations were found which were of any importance, but it appeared that an increase was required in the degree of some of the combinations already present.

However, in these computer results there began to appear indications that some statistically unsatisfactory feature was present in the analysis, and eventually this was traced to a group of 42 models which had extreme values of several parameters simultaneously. These models all had keel area ratios greater than 0.04 (no other models had), all had half-angle of run ($\frac{1}{2}\alpha r$) of 90° (only four other models had), all had high beam/draft (B/T) values (including all of the highest values) and very low values of buttock slope (α_{BS}) (including the very lowest values). Because of the distortion of the fitted equation likely to be caused in these circumstances, these 42 models were discarded. It turned out that the standard error of residuals was thereby reduced by a further 15 per cent.

Because of the change in the situation brought about by these omissions, the previous work, seeking new terms to add to the equation, was repeated with the reduced number of data: none of the terms tested now proved statistically significant - this change from the previous results being a further indication of the distortion produced by the 42 models.

Two minor modifications of the equation should be mentioned. One of these concerned the keel effect: the numerical results indicated that, rather than use a/Amax and its square, it was slightly preferable to use the original keel term, which allows a fixed increment when a keel of any size is fitted together with a/Amax only, which allows the keel effect to vary linearly with size of keel. This combination seems reasonable from a physical point of view.

The second modification concerned the tank blockage effect and was the omission of the $(B_1n)^2$ term. This was done because of trouble encountered when comparing estimates from the European equation with model test results. This occurs when the test is run in a tank with a lower cross-sectional area, relative to size of model, than that of the Swedish tank in which most of the data were obtained. This can result in B_1n being well outside the range encountered in the data, so that the computed correction can be grossly in error. The consequence of omitting the $(B_1n)^2$ term is that the blockage correction is not quite so good when working within the data range of this parameter, but it is likely to be much better when working outside the range (a situation which is undesirable but sometimes unavoidable).

Finally, when the results of the detailed re-examination of the data already mentioned had been completed, 11 further models were discarded because of special features or errors. The final standard error of residuals for a speed-length ratio of 1.0 was 1.08, or 4½ per cent of the average CR_{16} values.

Thus, the equation which gave this satisfactory result differed from the original European equation only in having the term a/Amax in place of $(B_1n)^2$; the difficulties previously encountered had been traced to various special features contained in the data.

In passing, it may be noted that the presence of a chine or of a cruiser stern had no apparent effect.

4. COMBINED DATA

Having reached a satisfactory position with the Japanese data, it was possible to reconsider the regression analysis for the combined data. In addition to the Japanese data (for 265 models) used in the final fitting of the last section, and the European data (for 276 models) used previously, there were available 16 model results recently supplied by Professor C. Ridgely-Nevitt (1967) and 13 results obtained from tests on models designed by FAO/NPL on the basis of the European regression equations, giving a total of 570 model results. When the form of equation used for the Japanese data was fitted to these data, the standard error of residuals for $V/\sqrt{L} = 1.0$ was 1.26 or 6 per cent of the average CR_{16} value. This was considered to be acceptable for the combined data.

However, before accepting this form of equation, which contained 86 terms, a detailed statistical examination of the various groups of terms included was carried out to assess whether any of them could be discarded as unnecessary. Previously, in building up the equation for the European data, and subsequently investigating modifications for the Japanese data, the emphasis had always been on adding new terms in order to improve the accuracy of fit. Moreover, in the case of European data, the numerical work had been carried out on the ACE computer at NPL, and the time required to carry out a comprehensive examination of the effect of all the various groups of terms (many of which had been included on qualitative physical arguments) would have been prohibitive. With the new programme written for the KDF 9 computer at NPL, however, such an examination could be undertaken.

Statistically speaking, the inclusion of a number of unnecessary terms in the equations does not matter very much, nor, if a computer is being used, is the evaluation of the equations for use in practice appreciably affected. Nevertheless, a comprehensive tabulation of the equation (see Chapter 6) would be simplified if the whole group of terms linking any particular pair of parameters could be omitted. Moreover, even if the whole group could not be omitted, the omission of any unnecessary higher-power terms is likely to improve the estimates given by the equation in parameter regions near the edges of the data.

The examination, which was carried out for $V/\sqrt{L} = 1.1$, showed that no complete group of terms could be omitted, but that a total of 14 of the higher-power terms could be. The standard error of residuals was not appreciably affected. The new form of equation containing 72 terms is contained in Appendix 1 and the corresponding numerical coefficients relevant to infinite water (i.e. $B_1n = 0$) in Appendix 3.

The number of models available at each speed-length ratio, and the resulting standard error of residuals, are given in Table I.

Table I

MODELS AVAILABLE AT EACH SPEED-LENGTH RATIO

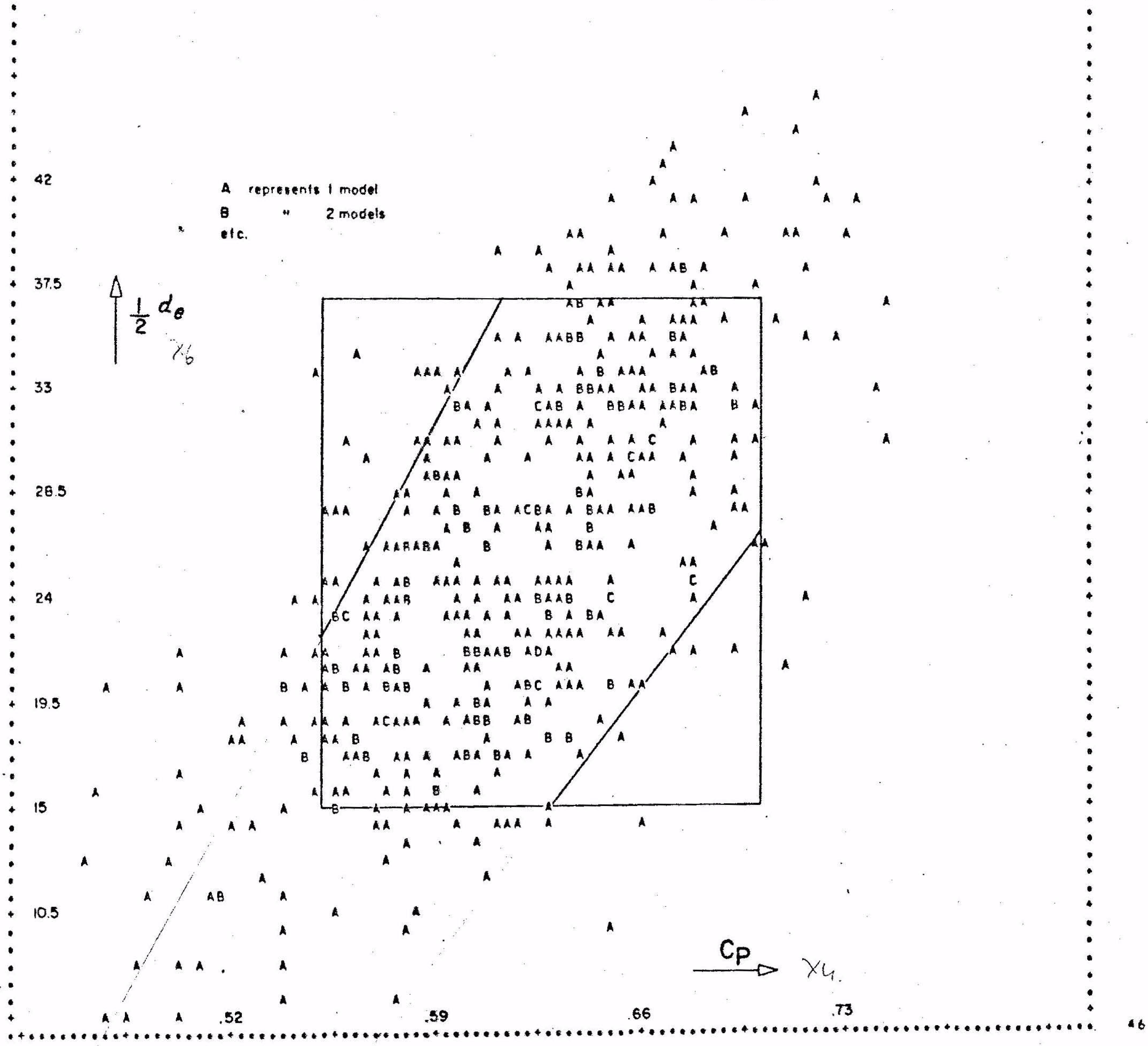
$V/\sqrt{L}$	0.90	0.95	1.00	1.05	1.10	1.15	1.20
Number of models	517	541	542	530	512	476	420
Standard error of residuals	1.13	1.14	1.23	1.36	1.47	1.51	1.60

5. REGIONS OF VALIDITY

Equations of the type derived are reliable only in subsequent practical application when they are used in parameter regions where there were sufficient data in the analyses. Outside such regions, the equations can be grossly in error. Since it involves the simultaneous consideration of at least the eight main parameters, the specification of these regions of validity is not easy to carry out rigorously. However, it is considered that a reasonably satisfactory procedure is first of all to lay down for each individual parameter a range which is sufficiently covered by data, and then to consider in turn the pairs of parameters which are interlinked in the equation.

An example is given in Fig. 1, which shows prismatic coefficient (Cp) plotted against half-angle of entrance ($\frac{1}{2}\alpha e$) for all the models in the data at $V/\sqrt{L} = 1.1$. The rectangle indicates the ranges laid down for the individual parameters, and the regression equation should be used only with great caution to estimate resistance when the combination of these two parameters gives a point outside the rectangle. Even

Fig. 1. Prismatic coefficient (Cp) plotted against half-angle of entrance (½ α_e).



within this rectangle data points are sparse in two of the corners. The system that has been used to define the region of validity is to draw straight lines cutting off these corners, the intention being that the regression equations should be used only in the remaining part of the rectangle.

This procedure was carried out for all the pairs of parameters linked together in the equations. The resulting set of inequalities, defining the complete region of validity at $V/\sqrt{L} = 1.1$, is given in Appendix 2. It is considered that these inequalities are applicable also to the other speed-length ratios. It may be observed that the positioning of the straight lines is not a clear-cut question since sparseness is a gradually varying phenomenon. The inequalities given must therefore be regarded as provisional; it must be left to future application of the equation to determine the limits more precisely.

6. TABULATION OF REGRESSION EQUATIONS

The regression equation (see Appendix 1), the inequalities in Appendix 2 and the coefficients in Appendix 3 give a complete specification of the results for practical application when a computer is available. However, to assist in the evaluation of the equation for $V/\sqrt{L} = 1.1$ without a computer, a comprehensive systematic tabulation of the equation is given in Appendix 4. A table is given for each of 107 combinations of L/B (length/beam) and B/T values. The upper part of that table gives the CR_{16} value for varying values of M (length-displacement ratio), Cp and $\frac{1}{2}\alpha e$, and standard values of the other parameters. The lower part of the table gives the increment to be added when the LCB (longitudinal centre of buoyancy) and αBS values differ from the standard.

Then after these 107 pairs of tables come four tables applicable to all values of L/B and B/T. The first of these outlines the increment to be added to the previously obtained CR_{16} value when $\frac{1}{2}\alpha r$ differs from the standard. The second gives an additional increment for LCB at different values of $\frac{1}{2}\alpha e$, and the last two tables give the increments concerned with trim and keel area, respectively.

In all the tables, the set of CR_{16} values, or increments which correspond to parameter combinations within the region of validity as defined in Chapter 5, have been boxed round for easy identification. Other values have been given because, from inspection, some of them may prove to be reasonable. Throughout, the tables apply to infinite water and fully stimulated turbulence.

7. EXAMPLES OF USE OF TABULATION

The tables of CR_{16} values presented in Appendix 4 can be used, in principle, for two kinds of calculation: evaluation of the resistance for given forms, and optimization of hull forms when only a few parameters are described.

In the examples below, no extrapolation has been made outside the ranges of validity (boxed areas in the tables and inequalities in Appendix 2). As pointed out in Section 5, these are not well-defined definite limits, and it might be that for some parameter combinations the values outside the established limits are as good as, or even better than, those inside them. In general, the uncertainty is greater when working outside the ranges and special caution is recommended in those cases.

Evaluation

Assume that a hull has the following parameters:

$L/B = 3.5$ $\quad B/T = 2.8$ $\quad Cm = 0.73$

$Cp = 0.613$ $\quad LCB = 1.0$ per cent $\quad \frac{1}{2}\alpha e = 30°$

$\frac{1}{2}\alpha_r = 60^\circ$	$\alpha_{BS} = 17^\circ$	trim = 0.03
$L = 78.7$ ft	$\Delta = 180$ tons	$S = 1\ 840\ ft^2$

The length-displacement ratio $M = L/\Delta^{1/3}$ is here 4.25. From the table for L/B = 3.5, B/T = 2.8, the following, valid for infinite water and fully turbulent flow, can be read, linearly interpolating as necessary.

The main table gives	CR_{16} =	18.81
LCB – α_{BS} correction		1.29
$\frac{1}{2}\alpha_r$ – Cp correction (last table page)		0.21
LCB – $\frac{1}{2}\alpha_e$ " "		– 0.43
Trim " "		– 0.11
No keel		–
		19.77

There are now innumerable ways in which the values of these parameters can be changed, still getting a craft of the same size. For simplicity, in this example it is assumed that, for various reasons, the displacement and main dimensions cannot be changed. What can then be done to decrease the resistance? A general look at the tables indicates that low Cp values and $\frac{1}{2}\alpha_e$ values are favourable and changes in these parameters are likely to give a substantial change in resistance.

When specifications and arrangements of the vessel are duly considered, it is found that the following changes can be made: Cp = 0.575 which will give a Cm = 0.777 (calculated from "Cm for Cp = 0.70" in the last column, giving 0.7×0.638/0.575. which derives from the relation $M = (L/B)^2\ (B/T)/(Cp.Cm)$, so that Cp x Cm is constant), LCB = 0, $\frac{1}{2}\alpha_e = 25^\circ$ and $\alpha_{BS} = 22^\circ$.

The main table then gives	CR_{16} =	16.89
LCB – α_{BS} correction		+ 0.44
$\frac{1}{2}\alpha_r$ – Cp " (last table page)		+ 0.56
LCB – $\frac{1}{2}\alpha_e$ " "		– 0.29
Trim and "no keel" unchanged		– 0.11
		17.49

Thus, the above rather modest alterations in the design give 11.5 per cent lower resistance.

The required towing power (EHP) can be calculated by the following formulae (using the ITTC friction coefficient).

$$EHP = \frac{CR_L\ \Delta V^3}{325.7\quad L}$$

where:

$$CR_L = CR_{16} - 0.212847 \left(\frac{S.L}{\Delta}\right) \left\{ \left[\log\left(88 \frac{V}{\sqrt{L}} \cdot 10^3\right)\right]^{-2} \left[\log\left(1.2834 \frac{V}{\sqrt{L}} L^{3/2} \cdot 10^3\right)\right]^{-2} \right\}$$

A full evaluation of the two above hull versions gives the following results:

Original

$V/\sqrt{L}$	0.9	0.95	1.00	1.05	1.10	1.15	1.20
CR_{16}	15.11	16.55	17.71	18.54	19.74	21.11	22.59
EHP	47	61	77	94	116	143	176

Modified

$V/\sqrt{L}$	0.9	0.95	1.00	1.05	1.10	1.15	1.20
CR_{16}	15.20	16.18	16.63	16.89	17.49	18.06	19.37
EHP	47	59	72	84	101	120	145

It should be noted that the tables in this report are calculated for a certain speed-length ratio (1.1) which means that the hulls must be of the same length to get a true comparison when based on CR_{16}. The speed-length ratio 1.1 has been selected for the printing of tables because it is considered that this is a speed at which most fishing vessels operate under normal service conditions. Although the actual resistance values are different for other speed-length ratios, the given tables roughly indicate the rate of variation in resistance at speeds not too far below or above speed-length ratio 1.1.

However, to estimate the power over a speed range or to compare hulls of different lengths at constant speed, the tables are not sufficient and the regression equation (see Appendix 1) with its coefficients (see Appendix 3) has to be used. Manual calculations are very tedious but the equation can easily be programmed for a computer.

Optimization

Questions such as "what is to be optimized" and "which parameters are variables and which are constant" are often not clearly answered in optimization problems and yet require careful consideration.

In our particular case the ultimate aim is usually to get a fishing vessel which is as profitable as possible. The thing to do then, and the best from a theoretical point of view, is to include the regression equations in a total techno-economic optimization model. This seems to be impractical, considering all the parameters and possible variations that affect only the resistance in a model which must also cover all other aspects of a fishing operation. It is believed that it is better to derive from the results of the regression study a general relation between resistance on the one hand and displacement, length and speed on the other, and to use that in a techno-economic model. The results from such a model would then be displacement, length and speed, and the resistance optimization becomes merely a "refinement" of the hull form or a "sub-optimization". For example, let us assume that the displacement (180 tons) and length (78.7 ft) of the previous hull are given, that there are no restraints on the other parameters and that the hull form is to be optimized or rather refined for a speed corresponding to speed-length ratio 1.1.

By scanning through the tables one finds that the lowest CR_{16} values appear in the L/B range 3.5 - 4.1 for B/T in the range 2.2 - 2.4. For these main dimensions the best Cp is 0.575, LCB is 4 per cent and $1/2\,\alpha_r$ is 30°. The best $1/2\,\alpha_e$ is the lowest within the range of validity (boxed area). The tables show that still lower resistance can be obtained if one goes outside the $1/2\,\alpha_e$ range. This might be correct but there are some values which are definitely too low and unreliable so once again it is safer to stay within the boxed area. Best α_{BS} varies with L/B.

Most of the CR_{16} values within the stated ranges where the actual optimum is located lie between 12.5 and 13. The differences between alternatives are so small that they might well be due to statistical errors in the regression functions and not to the variation in the parameters. One can therefore get an "optimum" hull without being restrained to accept one particular set of parameters.

The minimum CR_{16} value is found for the following parameters:

L/B = 3.9	B/T = 2.4	Cm = 0.828
Cp = 0.575	LCB = - 4 per cent	$1/2\,\alpha_e$ = 17.5°
$1/2\,\alpha_r$ = 30°	α_{BS} = 17°	

and the CR_{16} = 12.47

trim correction = -0.11 (not optimum but a practical design value)

12.36

This form has thus about 37 per cent lower resistance at speed-length ratio 1.1 than the "original" (see page 8).

The CR_{16} and EHP values over the full speed range are:

Optimized

$V/\sqrt{L}$	0.90	0.95	1.00	1.05	1.10	1.15	1.20
CR_{16}	11.23	11.38	11.50	11.73	12.36	12.79	12.82
EHP	33	39	46	55	68	81	92

Another optimization situation occurs when the displacement and speed are given but the length can be varied. In this case one would need tables for other speed-length ratios, which, with a fixed speed, will automatically provide CR_{16} values for different lengths. However, one can no longer compare resistance by using CR_{16} values directly since length is involved in their definition; instead one must use CR_L/L, where CR_L is obtained from the equation on page 8. Clearly such an optimization would be very laborious without the use of a computer.

Appendix 1

REGRESSION EQUATION

$$6 = a_0 + a_1X_1 + a_2X_2 + a_3X_3 + a_4X_4 + a_5X_5 + a_6X_6 + a_7X_7 + a_8X_8$$
$$+ a_9X_9 + a_{10}X_{10} + a_{11}X_{11} + a_{12}X_{12} + a_{13}X_{13}$$
$$+ a_{14}X_1^2 + a_{15}X_2^2 + a_{16}X_3^2 + a_{17}X_4^2 + a_{18}X_5^2 + a_{19}X_6^2 + a_{20}X_7^2 + a_{21}X_8^2$$
$$+ a_{22}X_1^3 + a_{23}X_2^3 + a_{24}X_3^3 + a_{25}X_4^3 + a_{26}X_5^3 + a_{27}X_6^3 + a_{28}X_8^3$$
$$+ a_{29}X_1^4 + a_{30}X_4^4 + a_{31}X_6^4$$
$$+ a_{32}X_1X_3$$
$$+ a_{33}X_1X_4 + a_{34}X_1^2X_4 + a_{35}X_1X_4^2 + a_{36}X_1^3X_4 + a_{37}X_1^2X_4^2 + a_{38}X_1X_4^3$$
$$+ a_{39}X_1X_6 + a_{40}X_1^2X_6 + a_{41}X_1X_6^2 + a_{42}X_1^3X_6 + a_{43}X_1^2X_6^2 + a_{44}X_1X_6^3$$
$$+ a_{45}X_1X_8 + a_{46}X_1^2X_8 + a_{47}X_1X_8^2$$
$$+ a_{48}X_2X_3 + a_{49}X_2^2X_3 + a_{50}X_2X_3^2$$
$$+ a_{51}X_2X_4 + a_{52}X_2^2X_4 + a_{53}X_2X_4^2$$
$$+ a_{54}X_2X_6 + a_{55}X_2^2X_6 + a_{56}X_2X_6^2$$
$$+ a_{57}X_2X_8$$
$$+ a_{58}X_4X_5 + a_{59}X_4^2X_5 + a_{60}X_4X_5^2$$
$$+ a_{61}X_4X_6$$
$$+ a_{62}X_4X_7$$
$$+ a_{63}X_4X_8 + a_{64}X_4^2X_8 + a_{65}X_4X_8^2$$
$$+ a_{66}X_5X_6 + a_{67}X_5^2X_6 + a_{68}X_5X_6^2$$
$$+ a_{69}X_5X_8 + a_{70}X_5^2X_8 + a_{71}X_5X_8^2$$

ere

X_1 = f (L/B)

X_2 = f (B/T)

X_3 = f (Cm)

X_4 = f (Cp)

X_5 = f (LCB)

X_6 = f ($\frac{1}{2}$ de)

X_7 = f ($\frac{1}{2}$ dr)

X_8 = f (dBS)

X_9 = f (trim)

X_{10} = f (B_1n)

X_{11} = f (turbulence stimulation)

X_{12} = f (wooden or bar keel)

X_{13} = f (a/Amax)

a_0, a_1, a_2, ...a_{71} are constants determined by the analysis, a different set for each speed-length ratio. Their numerical values and the full definition of the X's are given in Appendix III.

PARAMETER RANGES

$3.1 \le x_1 \le 5.6$ L/B

$2.0 \le x_2 \le 4.5$ B/T

$.53 \le x_3 \le .93$ Cx

$.55 \le x_4 \le .70$ Cp

$-6 \le x_5 \le +2$ Lcb

$15 \le x_6 \le 37$ αe

$30 \le x_7 \le 80$ er

$12 \le x_8 \le 32$ αbs

$-.04 \le x_9 \le .08$ trim

$x_{10} = 0$

$x_{11} = 0$

$x_{12} = 0$ or 1

$0 \le x_{13} \le .024$

$1.4x_1 - 9x_3 + 2.86 \ge 0$

$3x_1 - 16x_3 - 3.4 \le 0$

$7x_1 + 60x_4 - 56.8 \ge 0$

$2x_1 - 65x_4 + 37.3 \ge 0$

$7x_1 - 80x_4 + 10.8 \le 0$

$x_1 + 0.05x_6 - 4.25 \ge 0$

$15x_1 - 4x_6 - 12 \le 0$

$25x_1 + 2x_8 - 118 \ge 0$

$15x_1 - 4x_8 - 24 \le 0$

$13x_1 + x_8 - 91.8 \le 0$

$x_2 + 4.8x_3 - 5 \ge 0$

$0.3x_2 - x_3 - 0.55 \le 0$

$x_2 + 30x_4 - 24 \le 0$

$x_2 - 30x_4 + 13.5 \le 0$

$x_2 + 0.06x_6 - 3.4 \ge 0$

$8x_2 + x_6 - 61 \le 0$

$3x_2 + x_8 - 22.5 \ge 0$

$90x_2 + 11x_8 - 613.5 \le 0$

$230x_4 - x_6 - 104 \ge 0$

$160x_4 - x_6 - 85 \le 0$

$210x_4 - x_7 - 57 \ge 0$

$950x_4 - 6x_8 - 536 \le 0$

$3x_5 - x_6 + 45 \ge 0$

$3x_5 - x_6 + 18 \le 0$ →

$5x_5 - 9x_8 + 131 \le 0$

COEFFICIENTS

$V/\sqrt{L}$	0.90	0.95	1.00	1.05	1.10	1.15	1.20
Const.	22.570	25.282	27.595	30.965	33.326	36.422	37.726
x_1	8.552	9.526	11.013	14.380	16.557	18.086	18.134
x_2	9.474	10.815	11.587	13.189	13.582	17.883	18.500
x_3	-6.365	-6.526	-5.749	-5.891	-6.597	-6.152	-5.872
x_4	-1.904	-1.194	0.778	1.993	5.791	4.338	1.062
x_5	-1.818	-1.010	-0.718	-0.125	-0.655	0.763	2.075
x_6	2.306	2.985	4.942	7.060	10.413	12.815	17.585
x_7	0.598	0.505	0.350	0.437	0.097	0.374	1.016
x_8	4.091	5.174	5.171	6.704	5.481	7.757	8.529
x_9	0.297	0.653	0.921	0.688	0.494	0.421	0.841
x_{10}	0	0	0	0	0	0	0
x_{11}	0	0	0	0	0	0	0
x_{12}	0.665	0.711	0.953	0.720	0.159	0.416	0.147
x_{13}	2.177	2.225	2.557	3.540	4.224	3.566	2.505
x_1^2	-1.185	-2.238	-1.881	-0.905	1.877	4.670	1.547
x_2^2	1.703	0.232	1.777	4.916	7.625	4.381	6.128
x_3^2	2.307	3.307	3.821	3.891	4.658	3.703	4.116
x_4^2	-2.696	-2.441	-2.861	-2.215	-0.929	-1.233	-2.160
x_5^2	-3.317	-2.427	-1.173	-1.082	-1.624	-1.199	-1.450
x_6^2	-5.069	-7.023	-6.718	-5.484	-1.941	3.069	1.687
x_7^2	0.921	0.396	-0.344	-0.499	-0.611	-0.896	-1.604
x_8^2	1.220	1.546	1.226	1.594	0.206	-1.382	0.660
x_1^3	4.139	3.873	5.644	3.919	13.097	13.587	9.371
x_2^3	4.676	3.004	3.825	6.173	7.998	4.191	7.884
x_3^3	1.103	0.805	0.301	1.156	3.213	3.160	3.770
x_4^3	-3.290	-3.227	-4.681	-4.896	-6.923	-6.505	2.765
x_5^3	2.001	1.008	-0.169	-0.184	-0.097	-1.866	-3.359
x_6^3	-2.026	-2.512	-3.476	-3.864	-3.591	-6.606	-18.775
x_8^3	0.726	1.876	2.949	3.728	4.628	3.866	3.133
x_1^4	9.887	10.807	11.842	11.204	20.382	18.995	18.387
x_4^4	0.127	1.050	2.048	2.931	3.353	3.331	8.856
x_6^4	1.324	2.594	4.534	5.490	4.878	-0.716	-6.510
$x_1 x_3$	-7.863	-7.810	-6.821	-6.697	-7.527	-6.806	-4.271
$x_1 x_4$	-2.371	-2.289	-3.120	-3.851	-3.859	-5.821	-3.933
$x_1^2 x_4$	-10.511	-10.733	-14.530	-14.724	-32.762	-28.158	-13.483
$x_1 x_4^2$	0.277	0.152	-3.361	-10.159	-11.014	-11.185	-4.274

where:

x_1 = (L/B-4.75)/1.95

x_2 = (B/T-4.1)/2.6

x_3 = (Cm-0.715)/0.265

x_4 = (Cp-0.625)/0.155

x_5 = (LCB+3.25)/8.75

x_6 = (1/2 d e-28.5)/22.5

x_7 = (1/2 d r-52.5)/37.5

x_8 = (d BS-34)/25

x_9 = (trim-0.05)/0.09

x_{12} = 0 if no keel

= 1 if keel

x_{13} = (a/Amax-0.0202)/0.0202

$V/\sqrt{L}$	0.90	0.95	1.00	1.05	1.10	1.15	1.20
$x_1^3x_4$	-9.512	-10.136	-12.028	-10.074	-27.758	-20.894	-9.840
$x_1^2x_4^2$	-3.991	-3.127	-4.577	-15.566	-21.784	-24.728	-19.121
$x_1x_4^3$	6.421	7.634	4.936	3.257	1.888	4.318	9.820
x_1x_6	-0.601	-0.806	0.569	3.999	8.386	12.520	11.555
$x_1^2x_6$	5.156	1.645	4.818	4.441	15.961	33.566	16.100
$x_1x_6^2$	0.332	-0.663	0.815	1.374	11.698	21.602	9.909
$x_1^3x_6$	9.252	7.048	8.893	7.351	17.359	33.967	23.857
$x_1^2x_6^2$	1.516	-2.460	-0.699	-1.713	8.039	19.063	9.635
$x_1x_6^3$	5.365	6.031	7.671	7.471	11.020	12.966	0.220
x_1x_8	1.310	0.967	0.754	1.678	3.360	2.432	5.946
$x_1^2x_8$	-0.628	-2.306	-3.277	-3.563	-0.026	2.140	7.502
$x_1x_8^2$	1.575	0.982	-0.192	-1.063	-4.630	-9.855	-5.868
x_2x_3	-3.616	-4.515	-4.418	-4.349	-4.629	-4.303	-3.173
$x_2^2x_3$	-3.564	-4.646	-5.157	-6.226	-7.783	-8.344	-8.613
$x_2x_3^2$	0.244	2.071	2.762	3.623	3.842	1.642	3.077
x_2x_4	-5.293	-3.671	-3.966	-6.530	-7.951	-6.968	-8.687
$x_2^2x_4$	0.533	3.515	2.853	-2.743	-4.162	-0.321	-4.897
$x_2x_4^2$	-13.031	-11.918	-10.601	-11.019	-10.402	-14.066	-16.171
x_2x_6	3.852	1.665	5.744	10.642	17.134	15.606	17.743
$x_2^2x_6$	6.957	3.927	6.636	14.200	18.449	13.571	16.714
$x_2x_6^2$	4.655	2.740	7.338	10.116	15.274	14.476	14.659
x_2x_8	2.717	4.314	4.878	6.949	6.050	10.316	9.640
x_4x_5	-9.913	-10.255	-9.050	-7.599	-8.718	-6.870	-7.419
$x_4^2x_5$	-7.636	-6.862	-5.544	-6.856	-7.160	-6.025	-12.249
$x_4x_5^2$	10.930	10.227	10.016	9.537	9.370	10.824	6.044
x_4x_6	4.984	5.200	5.391	5.051	7.980	7.081	8.381
x_4x_7	-1.861	-2.238	-2.285	-1.717	-2.719	-2.037	-1.014
x_4x_8	-4.159	-5.673	-5.908	-8.251	-7.265	-5.178	-3.783
$x_4^2x_8$	3.032	4.812	6.070	6.372	7.151	6.947	0.451
$x_4x_8^2$	-4.129	-7.501	-9.539	-12.549	-11.695	-7.752	-0.850
x_5x_6	3.591	3.625	4.203	3.615	5.533	4.817	6.009
$x_5^2x_6$	-10.735	-7.734	-6.088	-5.306	-4.923	-5.925	-4.407
$x_5x_6^2$	8.393	8.245	12.109	13.950	16.272	14.780	22.057
x_5x_8	-0.521	1.379	3.077	4.524	4.312	6.178	5.999
$x_5^2x_8$	-8.420	-9.245	-9.874	-9.986	-11.067	-10.993	-12.033
$x_5x_8^2$	-0.136	1.614	3.576	4.167	4.700	6.431	5.515