The Bare Hull Resistance of the Delft Systematic Yacht Hull Series by

J.A. Keuning, R. Onnink, A. Versluis and A.A.M. van Gulik

1 Introduction

The data of the Delft Systematic Yacht Hull Series published so far have always been derived of experiments carried out with the hulls with their appendages, i.e. keel and rudder. Obviously to be able to perform tests with models of sailing yachts in the heeled and yawed conditions it is inevitable to add these appendages necessary for the side force production. Only in the upright condition it would make sense to carry out experiments with bare or unappended hulls. To shorten the time needed for the experiments on the original series only appended models were tested and for the sake of consistency throughout the series the physical dimensions of the keel and rudder, used during the tests, have not changed throughout the testing of the entire series, which spans a period in time now of roughly 25 years, i.e. from 1974 to 1996.

During this period the shape of the appendages under actual sailing yachts and racing yachts in particular has changed dramatically. Not only the shape of the keel and rudder but also their volume related to the overall displacement of the yacht changed considerably over the last decades. As a typical demonstration of this a comparison between the parent model of Series 1, being derived from the well known Standfast 43 designed by Frans Maas, and a modern sport boat type is shown in Figure 1. The difference in keel design is obvious.

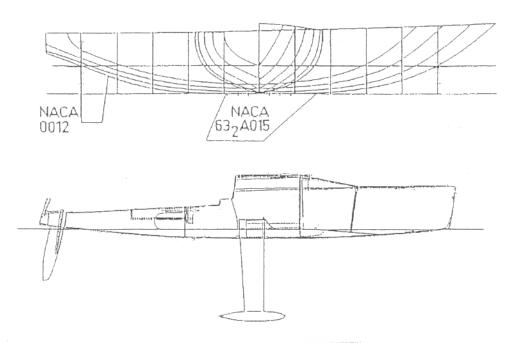


Figure 1: Comparison hull shape old and modern yacht

Yet when an approximation method for the resistance of the yachts derived from the data of the DSYHS is used the volume of the keel and rudder as used in the DSYHS is supposed to be present, since all data on the residuary resistance are on the appended hull. To get some insight in

the relative importance of this keel and rudder volume as well as wetted area related to the overall displacement and wetted area, Figure 2 and 3 have been prepared.

KEEL AND RUDDER / HULL DISPLACEMENT

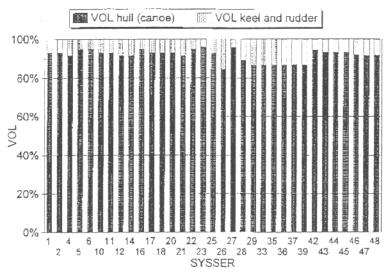


Figure 2: Relative keel and rudder / hull displacement

KEEL AND RUDDER / HULL WETTED SURFACE

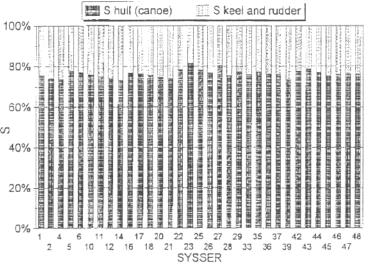


Figure 3: Relative keel and rudder / hull wetted surface

From these figures it may be seen that the contribution of the keel to the overall volume of displacement varies considerably over the range of models tested and may go as high as 17% for the lighter models. The difference in wetted area contribution of the appendages to the overall is much less pronounced; generally spoken the contribution for the light and heavy models is in the region of 25%.

How considerable the contribution of the appendages to the overall resistance may be is clearly demonstrated in the Figures 4 and 5.

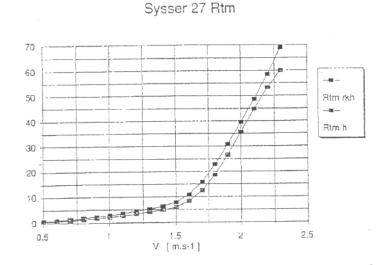


Figure 4: Comparison resistance with and without keel model of Sysser 27

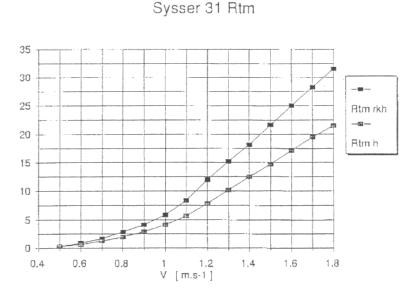


Figure 5: Comparison resistance with and without keel model of Sysser 31

Herein the resitance of the bare hull and the appended hull of a heavy and a very light displacement hull of the DSYHS is compared as measured in the towing tank. As may be seen the contribution may get up as high as roughly 30% on the overall resistance. This was considered to be a too big a constriction on the general applicability of the results of the DSYHS. Therefore it was decided to test a large number of the 40 models of the original Series again but now without appendages to obtain the resistance data of the bare hulls alone. Obviously the data is restricted to the upright resistance condition only.

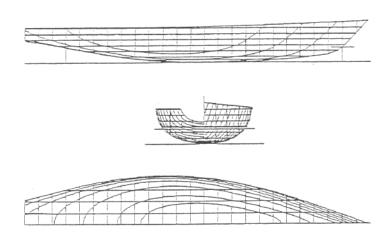
This testing of the bare hulls started in the Delft Shiphydromechanics Laboratory of the Delft University of Technology in 1991 and the first results are presented in this paper because now a large number of hulls of the DSYHS have been re-tested to yield sufficient data for a reliable regression analysis.

Since the resistance of the bare hull may now be approximated using the derived polynomials it becomes necessary to "add the appendages back on". This has to be done for a wide variety of different keel geometries under a wide variety of different hulls, which has proven to be a considerable problem. Possible approaches to this problem and to the hull - keel interaction have been (and will be) addressed in separate papers.

2 Models and Measurement Setup

The models used for the present experiment for determining the bare hull resistance are models originally belonging to the Delft Systematic Yacht Hull Series, as previously tested and reported. Not all of the models of the original DSYHS have (and will be) tested without their appendages.

To update the hullshape of the models belonging to the DSYHS as much as possible, a new parent model was introduced to the Series in 1994. The lines of this model were kindly put to our disposal by the well known design office of Sparkman and Stephens in New York (USA) and were originally used by Dr. P. Sclavounos from MIT, Boston (USA) to evaluate the added resistance in waves of IMS racing yachts using his 3-D code SWAN. The lines of this new parent model are depicted in Figure 6. The main particulars of this model are presented in Table 1. The series of systematic models derived from this new parent model are generally referred to as "Series 4".



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Figure 6: Parent model by Sparkman and Stephens, New York (USA)

7	#	L_{wl}/B_{wl}	B_{wl}/T_c	$L_{wl}/\Delta_c^{1/3}$	LCB	LCF	C_b	C_p	C_w	C_m
[-	-]	[-]	[-]	[-]	[%]	[%]	[-]	[-]	[-]	[-]
4	6	3.319	5.569	5.379	-3.290	-6.260	0.394	0.553	0.668	0.712

Table 1: Main Particulars Parent model by Sparkman and Stephens, New York (USA)

For the sake of comparison also the lines plans of the Parent model #1 of "Series 1" and of Parent model #2 of "Series 2" and "Series 3" together with their main particulars are presented in Figures 7 and 8 and Tables 2 and 3.

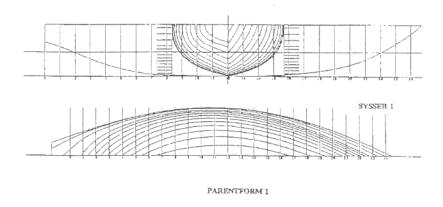


Figure 7: Parent 1 of series 1

#	L_{wl}/B_{wl}	B_{wl}/T_c	$L_{wl}/\Delta_c^{1/3}$	LCB	LCF	C_b	C_p	C_w	C_m
[-]	[-]	[-]	[-]	[%]		[-]	[-]	[-]	[-]
1	3.155	3.992	4.775	-2.290	-3.330	0.365	0.564	0.688	0.646

Table 2: Main Particulars Parent model Series 1

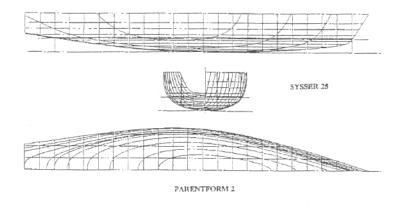


Figure 8: Parent 1 of series 2 and 3

The difference between in particular Series 1 and Series 2,3,4 with respect to cross sectional shape and aft overhang is quite significant. The influence of the overhangs is taken into account implicitly in the data since only the geometrical waterline length at zero forward speed is used as a length parameter. The total list of models which have been tested without their appendages is presented in Table 4 together with their main parameters. The typical model size of the models of Series 1, i.e. model #1 to model #22, was 1.65 meter waterline length and for the Series 2, 3 and 4, i.e. model #23 to model #48, this length was 2.00 meter.

The models were tested in the #1 towing tank of the Laboratory. This towing tank is 145 meters

	#	L_{wl}/B_{wl}	B_{wl}/T_c	$L_{wl}/\Delta_c^{1/3}$	LCB	LCF	C_b	C_p	C_w	C_m
	[-]	[-]	[-]	[-]	[%]	[%]	[-]	[-]	[-]	[-]
ľ	25	4.000	5.388	6.003	-1.990	-5.540	0.399	0.548	0.671	0.727

Table 3: Main Particulars Parent model Series 2 and Series 3

long, 4.5 meters wide and has a maximm waterdepth of 2.5 meters. The carriage on this tank is capable of speeds up to $8m.s^{-1}$.

The fact that all the new tests were now performed in this large towingtank made it possible to overcome one of the drawbacks of the original DSYHS, i.e. the fact that model #1 to model #22 were all tested with speeds up to $F_n = 0.45$, which corresponds to the maximum speed possible in the smaller #2 towing tank of the Laboratory and the models #22 to #39 with speeds upto a Froude number corresponding to $F_n = 0.70$, because these were already tested in this larger #1 tank. This difference in speed regime necessitated from the beginning the use of two seperate polynomials, i.e. one for the "low" speed range, i.e. $0 < F_n < 0.45$ and one for the "high" speed range, i.e. $0.45 < F_n < 0.70$. But now all bare hulls have been tested to speeds as high as physically meaningfull, but at least to a speed corresponding to $F_n = 0.60$ and the lighter models even higher, i.e. to speeds upto $F_n = 0.75$. The upper speed limit was generally imposed on the experiment by excessive wave generation and associated problems.

The standard method of turbulence stimulation of the Laboratory has been used during the experiments, i.e. three full width strips of carborundum grains evenly distributed along the length of the forward part of the canoe body of the model. In order to be able to correct for the resistance of the carborundum strips themselves all the tests have been carried out twice: once with half width and one with full width of the strips. Twice the difference between these two measured resistances (i.e. the resistance of the half width strip) is averaged over a speed range between $0.15 <= F_n <= 0.40$ and subtracted from the measurements to obtain the actual total resistance of the hulls.

The measurement setup was identical to the setup used in all previous experiments with the models of the DSYHSThe standard measurement setup of the Laboratory has been used. The models were connected to the towing carriage in such a way that they were free to heave, pitch and roll but restrained in all other modes of motion. The forces on the model were measured using strain gauge type dynamometers. All testruns had a measurement duration of at least 20 seconds and all forces were determined as an average over this period. The speed of the towing carriage is automatically controlled during the run within plus or minus $0.001m.s^{-1}$ from the preset speed.

During the tests reported here no additional speed dependent trimming moment due to the forces of the (imaginary) sails was applied. Identical tests, in which this trimming moment was applied, are not reported here.

3 Test Results

All the data of the experiments have been extrapolated to full scale yachts with an identical water-line length of 10.00 meters. The well known Froude extrapolation method has been used for this procedure. The ITTC-57 friction line has been used for the determination of the hull friction.

$$C_f = 0.075/(logR_e - 2)^2 \tag{1}$$

For the determination of the Reynolds number of the hull $0.9*L_{wl}$ has been used.

In order to investigate the possible magnitude of the formfactor k the well known procedure as described by Prohaska has been used. From these plots it became obvious that in practically all models the form factor k is small, typically between 0.03 and 0.05. A few examples of these plots are presented in Figure 9.

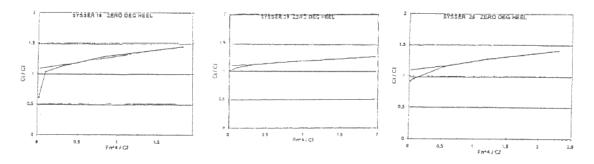


Figure 9: Example of Prohaska plots

Based on the results obtained from the determination of the form factor k using these Prohaska plots it was decided not to use any form factor in the extrapolation procedure of the full scale results. This is consistent with all previous work of the DSYHS and it actually extends the general applicability of the DSYHS results, because every time it turns out to be impossible to find an expression which predicts with sufficient accurracy the form factor of an arbitrary hull shape as function of the hull geometry parameters used. To approximate the full scale resistance of an arbitrary yacht using a polynomial expressions based on data of an extrapolation to full scale using a formfactor would make this a necessity however.

4 Polynomial Expression

The aim of the present study is to derive a polynomial expression for the residuary resistance of the bare hull of an arbitrary sailing yacht. This expression should be capable to approximate this resistance component for a wide variety of yacht hulls geometries, i.e. "old" and "new" as well as "racing" and "cruising", with sufficient accurracy in order to be used in a "designers" VPP environment. This means that it has to predict the trends in resistance change with change of geometry descriptive parameters under consideration correctly. The starting point of the procedure is to use parameters in the regression analysis only which have a physical meaning to the problem. It is well known to anyone familiar with the procedures of applying regression analysis on data in order to obtain the coefficients of a polynomial expression to represent the original data, that a variety of possible expressions may be used, each with their own particular "pro's and con's". So for each particular application of the derived polynomial expression a "best" solution may exist. From various research projects it turned out that a reliable method is found by determining the "specific residuary resistance", i.e. the residuary resistance nondimensionalized by dividing it by the weight of displacment of the hull, and formulating polynomial expressions for its approximation at regular spaced Froude numbers.

One of the obvious restrictions for the formulation of the polynomial expression lies in the variations used in the DSYHS itself: the parameters varied in the series are limited. In Series 1 the variables

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were: the length **orbYachttDesign and YachtdGonstruction** the length displacement ratio (L/∇) , the prismatic coefficient (C_p) and the longitudinal position of the centre of buoyancy (LCB). This resulted in a polynomial expresssion as published by Gerritsma e.a. [1] of the following form:

$$\frac{R_R}{\Delta_C} 10^3 = A_0 + A_1 C_p + A_2 C_p^2 + A_3 LCB + A_4 LCB^2 + A_5 \frac{B_{wl}}{T_c} + A_6 \frac{L_{wl}}{\nabla_c^{1/3}}$$
(2)

The speed range in which it was applicable was $0.125 < F_n < 0.450$.

After introduction of Series 2 and 3 with the inherent problem of the different speed range for these Series, the following expressions were introduced by Gerritsma e.a. [1]:

For speed range $0.125 \le F_n \le 0.450$:

$$\frac{R_R}{\Delta_C} 10^3 = a_0 + a_1 C_p + a_2 LCB + a_3 \frac{B_{wl}}{T_c} + a_4 \frac{L_{wl}}{\nabla_c^{1/3}} + a_5 C_p^2 + a_6 C_p \frac{L_w l}{\nabla_c^{1/3}} + a_7 LCB^2 + a_8 (\frac{L_w l}{\nabla_c^{1/3}})^2 + a_9 (\frac{L_w l}{\nabla_c^{1/3}})^3$$
(3)

For speed range $0.475 <= F_n <= 0.750$:

$$\frac{R_R}{\Delta_C} 10^3 = c_0 + c_1 \frac{L_{wl}}{B_{wl}} + c_2 \frac{A_w}{\nabla_c^{2/3}} + c_3 LCB + c_4 \left(\frac{L_{wl}}{B_{wl}}\right)^2 + c_5 \frac{L_{wl}}{B_{wl}} \left(\frac{A_w}{\nabla_c^{2/3}}\right)^3 \tag{4}$$

The most striking difference with the original polynomial was found in the coupling of Cp with the length displacement ratio and the higher order terms for the length displacement ratio in the "low" speed polynomial and some terms originating from the planing boat world, like the "loading factor", which relates the weight of displacement to the planing area of the hull $(Aw/\Delta^{2/3})$ and the introduction of the length beam ratio (L/B) in the "high" speed polynomial.

A principal difficulty with the use of these two separate polynomial expressions was found in the "connection" at $F_n = 0.45$, were a stepless transition between the two polynomials was not always present and the resistance curve consequently had to be "faired" to yield one smooth curve.

Based on the experience gained with the previous expressions and enabled by the present existence of one consistent data base for all bare hull boats, a number of polynomial expressions to fit all" speeds upto $F_n = 0.60$ were formulated. For all of these the coefficients have been determined using a least square fit regression analysis tool. The final selection between the different expressions has been made on basis of the closest fit to the data, the proper prediction of trends and the robustness of the expression when used "at the corners" of the parameter space of the original models. Finally one expression was selected which is presented here. It is worth mentioning that the final selection inevitably contains a subjective component. The principal differences between the here presented expression with the previous ones are:

- All parameters are coupled with the displacement to length ratio
- The parameters are presented in such a way that their supposed contribution to the residuary resistance has the same trend as the displacement to length ratio with which they are coupled
- · The beam to length ratio is introduced again for all speeds

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- The beam ton Yachto Design and Yachto Construction lacement and the wetted surface of the canoebody, i.e. $\nabla^{1/3}/S_c$. This parameter amongst other things is considered to be more robust than the beam to draft ratio as used previously
- The LCB LCF spacing is introduced as a measure of possible hull "distortion" both fore and aft.
- ullet Both LCB and C_{p} are introduced to the second order

The polynomial expression used reads:

$$\frac{R_r}{\nabla_c 10^3 g} = a_0 + \left(a_1 \frac{LCB_{fpp}}{L_{wl}} + a_2 C_p + a_3 \frac{\nabla_c^{2/3}}{A_{wl}} + a_4 \frac{B_{wl}}{L_{wl}} \right) \frac{\nabla_c^{1/3}}{L_{wl}} + \left(a_5 \frac{\nabla_c^{2/3}}{S_c} + a_6 \frac{LCB_{fpp}}{LCF_{fpp}} + a_7 \left(\frac{LCB_{fpp}}{L_{wl}} \right)^2 + a_8 C_p^2 \right) \frac{\nabla_c^{1/3}}{L_{wl}} \tag{5}$$

in which:

R_R	Residuary Resistance Canoe Body	[N]
L_{wl}	Length on waterline	[m]
B_{wl}	Beam on Waterline	[m]
C_p	Prismatic Coefficient	[-]
∇_c	Volume of Displacement Canoe Body	$[m^3]$
LCB_{fpp}	Length center of Buoyancy measured from fore perpendicular	[m]
LCF_{fpp}	Length Center of Floatation measured from fore perpendicular	[m]
A_{wl}	Area of Waterline Surface	$[m^2]$
S_c	Area of Wetted Surface Canoe Body	$[m^2]$
g	Gravitational constant	$(9.81m.s^{-2})$

The range of applicability is $0.125 \le F_n \le 0.600$. T You can find the coefficients a_0 to a_8 for several F_n in Table 5.

For the sake of comparison the polynomial approximation according to expression 2 has been fitted to the bare hull data also in order to establish whether expression 3 really improved the prediction. In Figure 10, 11, 12 and 13 some typical results of the polynomial approximation according to expression #3 to the model tests results is shown. As may be concluded from these figures the fit of expression #3 to the model tests results is generally spoken very good. This conclusion holds true for all models belonging to the data set.

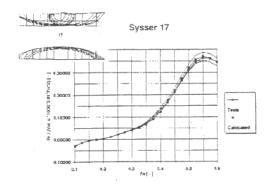


Figure 10: Comparison results model tests with expression 5 Sysser 17

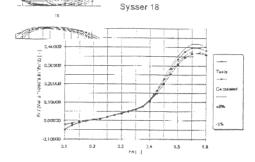


Figure 11: Comparison results model tests with expression 5 Sysser 18

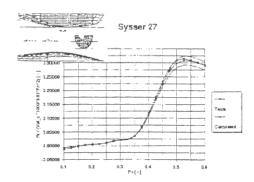


Figure 12: Comparison results model tests with expression 5 Sysser 27

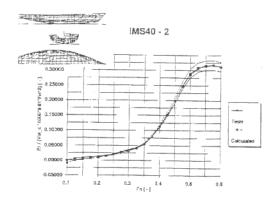


Figure 13: Comparison results model tests with expression 5 Sysser 42

For most of the models there is a somewhat improved correlation with the data using expression #3 when compared with the approximation according to expression #2. In some particular cases like the one shown in Figure 14 the improvement is considerable.

Of more interest however is a comparison of the approximation with measured results of models not belonging to the original data set in order to be able to check the general applicability of the presented expressions. Therefore Figure 15 and Figure 16 have been compiled as a typical example of a larger number of similar examples of correlation with some seven modeltests with unappended hulls carried out earlier at the Delft Ship Hydromechanics Laboratory.

For these two different hulls under consideration, both derived from the same parent but with a quite different beam to draft (B/T_c) and length to beam (L/B) ratio but with identical length displacement ratio, the residuary resistance has been calculated according to expression #2 and #3

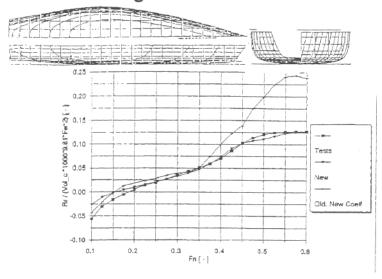


Figure 14: Comparison expression 5 with expression 3 and expression 4

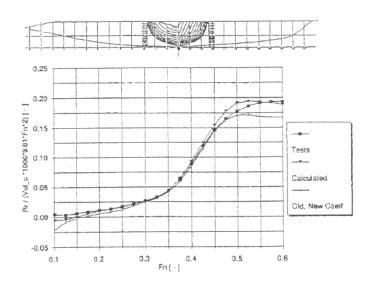


Figure 15: Comparison expression 5 with expression 3 and expression 4 and with model tests

and is compared in the figures to the similar values obtained from a towing tank experiment at the Laboratory. As may be seen from these results the correlation of expression #3 with the measured data is quite good and in particular at the higher speeds the correlation is improved over the one obtained with expession #2. In Figure 16 the discontinuity in expression #2 is clear.

5 Conclusion

Introduction of the bare hull resistance data enables a better prediction of the resistance of sailing yachts with a large variety in hull and appendage shapes. From the results presented in this paper it may be concluded that a fair approximation of the upright resistance of the unappended hull of a sailing yacht is possible using a polynomial expression of the given form. Different expression however may be more suitable for different applications. Extension on the database will remain a valuable goal to strive for.

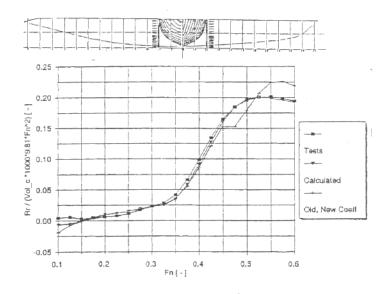


Figure 16: Comparison expression 5 with expression 3 and expression 4 and with model tests

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#	L_{wl}/B_{wl}	B_{wl}/T_c	$L_{wl}/\Delta_c^{1/3}$	LCB	LCF	C_b	C_p	$\cdot C_w$	C_m
[-]	[-]	[-]	[-]	[%]	[%]	[-]	[-]	[-]	[-]
1	3.155	3.992	4.775	-2.290	-3.330	0.365	0.564	0.688	0.646
2	3.623	3.043	4.776	-2.300	-3.340	0.367	0.567	0.691	0.646
3	2.747	5.345	4.779	-2.300	-3.320	0.370	0.572	0.695	0.647
4	3.509	3.947	5.097	-2.290	-3.330	0.367	0.568	0.691	0.646
5	2.747	3.957	4.356	-2.410	-3.430	0.361	0.559	0.683	0.647
6	3.155	2.979	4.339	-2.400	-3.420	0.363	0.561	0.685	0.646
7	3.155	4.953	5.143	-2.290	-3.350	0.362	0.561	0.685	0.646
8	3.279	3.841	4.775	-2.400	-3.320	0.379	0.586	0.707	0.647
9	3.049	4.131	4.776	-2.200	-3.340	0.353	0.546	0.672	0.646
10	3.155	3.992	4.775	0.000	-1.910	0.365	0.564	0.694	0.646
11	3.155	3.992	4.775	-4.980	-4.970	0.365	0.565	0.682	0.646
12	3.509	3.936	5.104	-0.010	-1.930	0.364	0.564	0.693	0.647
13	3.509	3.936	5.104	-5.010	-5.010	0.364	0.564	0.681	0.646
14	3.509	3.692	5.104	-2.300	-3.470	0.342	0.529	0.657	0.646
15	3.165	3.683	4.757	-2.290	-3.450	0.343	0.530	0.658	0.646
16	3.155	2.810	4.340	-2.300	-3.480	0.342	0.529	0.657	0.646
17	3.155	4.244	4.778	-0.010	-1.790	0.387	0.598	0.724	0.647
18	3.155	4.244	4.778	-5.000	-4.890	0.387	0.599	0.712	0.647
19	3.155	3.751	4.777	0.010	-2.060	0.342	0.530	0.664	0.646
20	3.155	3.751	4.778	-4.990	-5.090	0.342	0.530	0.651	0.646
21	3.509	4.167	5.099	-2.290	-3.220	0.387	0.598	0.718	0.647
22	2.732	4.231	4.337	-2.290	-3.220	0.387	0.599	0.719	0.647
23	3.472	4.091	5.001	-1.850	-5.290	0.394	0.547	0.673	0.721
24	3.497	10.958	6.935	-2.090	-5.840	0.402	0.543	0.670	0.739
25	4.000	5.388	6.003	-1.990	-5.540	0.399	0.548	0.671	0.727
26	3.994	12.907	7.970	-2.050	-6.330	0.407	0.543	0.678	0.749
27	4.496	2.460	5.011	-1.880	-5.240	0.395	0.546	0.677	0.724
28	4.500	6.754	6.992	-2.050	-5.950	0.400	0.544	0.672	0.736
29	4.000	10.870	7.498	-4.590	-7.630	0.413	0.549	0.671	0.751
30	4.000	7.082	6.500	-4.560	-7.660	0.413	0.549	0.672	0.751
31	4.000	15.823	8.499	-4.530	-7.810	0.412	0.548	0.674	0.752
32	4.000	10.870	7.498	-2.140	-6.220	0.413	0.549	0.687	0.751
33	4.000	10.870	7.498	-6.550	-8.730	0.413	0.549	0.659	0.751
34	4.000	10.373	7.491	-4.370	-7.550	0.395	0.522	0.649	0.757
35	4.000	11.468	7.472	-4.490	-7.580	0.440	0.580	0.694	0.758
36	4.000	10.163	7.470	-4.360	-7.290	0.390	0.551	0.663	0.707
37	4.000	9.434	7.469	-4.420	-6.930	0.362	0.552	0.654	0.657
38	3.000	19.378	7.503	-4.530	-7.860	0.413	0.547	0.675	0.755
39	5.000	6.969	7.499	-4.550	-7.540	0.413	0.549	0.670	0.753
41	4.000	5.208	5.927	-8.160	-9.510	0.400	0.540	0.652	0.741
42	3.319	3.711	4.699	-3.280	-6.410	0.394	0.554	0.670	0.711
43	2.784	6.291	4.983	-3.280	-6.490	0.394	0.553	0.672	0.712
44	3.319	4.424	4.982	-3.290	-6.250	0.394	0.554	0.668	0.712
45	4.175	2.795	4.982	-3.280	-6.240	0.394	0.554	0.668	0.711
46	3.319	5.569	5.379	-3.290	-6.260	0.394	0.553	0.668	0.712
47	3.337	6.042	5.474	-6.020	-8.400	0.410	0.548	0.699	0.749
48	3.337	5.797	5.426	-0.650	-5.030	0.404	0.557	0.690	0.725

Table 4: Main Particulars of all models used

F_n	0.10	0.15	0.20	0.25	0.30	
a_0	-0.00086	0.00078	0.00184	0.00353	0.00511	
a_1	-0.08614	-0.47227	-0.47484	-0.35483	-1.07091	
a_2	0.14825	0.43474	0.39465	0.23978	0.79081	
a ₃	-0.03150	-0.01571	-0.02258	-0.03606	-0.04614	
a_4	-0.01166	0.00798	0.01015	0.01942	0.02809	
a ₅	0.04291	0.05920	0.08595	0.10624	0.10339	
a_6	-0.01342	-0.00851	-0.00521	-0.00179	0.02247	
a_7	0.09426	0.45002	0.45274	0.31667	0.97514	
a_8	-0.14215	-0.39661	-0.35731	-0.19911	-0.63631	
-						,
F_n	0.35	0.40	0.45	0.50	0.55	0.60
F_n a_0	0.35 0.00228	-0.00391	0.45 -0.01024	0.50 -0.02094	0.55 0.04623	0.60 0.07319
-						
<i>a</i> ₀	0.00228	-0.00391	-0.01024	-0.02094	0.04623	0.07319
a_0 a_1	0.00228 0.46080	-0.00391 3.33577	-0.01024 2.16435	-0.02094 7.77489	0.04623 2.38461	0.07319 -2.86817
$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$	0.00228 0.46080 -0.53238	-0.00391 3.33577 -2.71081	-0.01024 2.16435 -1.18336	-0.02094 7.77489 -7.06690	0.04623 2.38461 -6.67163	0.07319 -2.86817 -3.16633
$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}$	0.00228 0.46080 -0.53238 -0.11255	-0.00391 3.33577 -2.71081 0.03992	-0.01024 2.16435 -1.18336 0.21775	-0.02094 7.77489 -7.06690 0.43727	0.04623 2.38461 -6.67163 0.63617	0.07319 -2.86817 -3.16633 0.70241
a_0 a_1 a_2 a_3 a_4	0.00228 0.46080 -0.53238 -0.11255 0.01128	-0.00391 3.33577 -2.71081 0.03992 -0.06918	-0.01024 2.16435 -1.18336 0.21775 -0.13107	-0.02094 7.77489 -7.06690 0.43727 0.11872	0.04623 2.38461 -6.67163 0.63617 1.06325	0.07319 -2.86817 -3.16633 0.70241 1.49509
a_0 a_1 a_2 a_3 a_4 a_5	0.00228 0.46080 -0.53238 -0.11255 0.01128 -0.02888	-0.00391 3.33577 -2.71081 0.03992 -0.06918 -0.39580	-0.01024 2.16435 -1.18336 0.21775 -0.13107 -0.34443	-0.02094 7.77489 -7.06690 0.43727 0.11872 -0.14469	0.04623 2.38461 -6.67163 0.63617 1.06325 2.09008	0.07319 -2.86817 -3.16633 0.70241 1.49509 3.00561

Table 5: Coefficients for expression 5